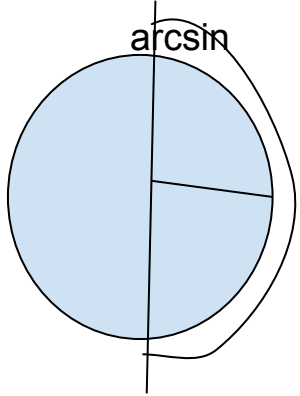


$$\int \sqrt{a^2 - x^2} dx$$



$$\begin{aligned} \int \sqrt{a^2 - x^2} dx &= [x = a \sin t] = \int \sqrt{a^2 - (a \sin t)^2} d(a \sin t) = \\ &= \int \sqrt{a^2 - a^2 \sin^2 t} d(a \sin t) = \int \sqrt{a^2 (1 - \sin^2 t)} d(a \sin t) = \\ &= \int a \sqrt{1 - \sin^2 t} d(a \sin t) = \int a \cos t d(a \sin t) = \\ &= \int a \cos t \cdot a \cos t dt = \int a^2 \cos^2 t dt = \\ &= \int a^2 \frac{1 + \cos 2t}{2} dt = \frac{a^2}{2} \int (1 + \cos 2t) dt = \\ &= \frac{a^2}{2} \left( t + \frac{\sin 2t}{2} \right) + C = \\ &= \frac{a^2}{2} \left[ \arcsin\left(\frac{x}{a}\right) + \frac{1}{2} \sin 2\left(\arcsin\left(\frac{x}{a}\right)\right) \right] + C = \\ &= \frac{a^2}{2} \left[ \arcsin\left(\frac{x}{a}\right) + \frac{1}{2} \cdot 2 \left(\frac{x}{a}\right) \sqrt{1 - \left(\frac{x}{a}\right)^2} \right] + C = \\ &= \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right) + \frac{1}{2} \cdot 2 \left(\frac{x}{a}\right) \sqrt{a^2 - \left(\frac{x}{a}\right)^2} + C \\ &= \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right) + (0.5)(x) \sqrt{a^2 - x^2} + C \end{aligned}$$

$$\begin{aligned} \sin[2(\arcsin(x/a))] &= \sin[2u] = 2 \sin u \cos u = 2 \left(\frac{x}{a}\right) \sqrt{1 - \left(\frac{x}{a}\right)^2} \\ (\arcsin(x/a)) &= u \quad | \quad \sin \quad (u \in [-\pi/2; \pi/2]) \\ (x/a) &= \sin u \\ \cos u &= \pm \sqrt{1 - \sin^2 u} = \pm \sqrt{1 - \left(\frac{x}{a}\right)^2} = \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad (\cos u > 0) \end{aligned}$$

$$\begin{aligned} \sin[2 \arccos y] &= \sin[2u] = 2 \sin u \cos u = \\ &= 2y \sqrt{1 - y^2} \\ \arccos y &= u \quad | \quad u \in [0; \pi] \\ y &= \cos u \\ \sin u &= \pm \sqrt{1 - \cos^2 u} = \sqrt{1 - \cos^2 u} = \\ &= \sqrt{1 - y^2} \end{aligned}$$

$$\begin{aligned} \sin[\arctg y] &= \sin[u] = \\ \arctg y &= u \quad | \quad u \in [-\pi/2; \pi/2] \\ y &= \tg u \\ 1 + \tg^2 u &= \frac{1 + \sin^2 u}{\cos^2 u} = \\ &= \frac{1}{\cos^2 u} = \frac{1}{1 - \sin^2 u} \\ 1 + y^2 &= \frac{1}{1 - \sin^2 u} \\ \frac{1}{1 + y^2} &= 1 - \sin^2 u \\ \sin u &= \pm \sqrt{1 - \frac{1}{1 + y^2}} \end{aligned}$$