

267. Примеры. $\int (6x^2 - 3x + 5) dx.$

Пользуясь правилами II и I (и формулами 3, 2), имеем

$$\begin{aligned} \int (6x^2 - 3x + 5) dx &= \int 6x^2 dx - \int 3x dx + \int 5 dx = \\ &= 6 \int x^2 dx - 3 \int x dx + 5 \int dx = 2x^3 - \frac{3}{2} x^2 + 5x + C. \end{aligned}$$

$$\begin{aligned} \int (2x^2 + 1)^3 dx &= \int (8x^6 + 12x^4 + 6x^2 + 1) dx = \\ &= \frac{8}{7} x^7 + \frac{12}{5} x^5 + 2x^3 + x + C. \end{aligned}$$

$$\begin{aligned} \int (1 + \sqrt{x})^4 dx &= \int (1 + 4\sqrt{x} + 6x + 4x\sqrt{x} + x^2) dx = \\ &= \int dx + 4 \int x^{\frac{1}{2}} dx + 6 \int x dx + 4 \int x^{\frac{3}{2}} dx + \int x^2 dx = \\ &= x + \frac{8}{3} x^{\frac{3}{2}} + 3x^2 + \frac{8}{5} x^{\frac{5}{2}} + \frac{1}{3} x^3 + C \end{aligned}$$

$$\begin{aligned} \int \frac{(x+1)(x^2-3)}{3x^2} dx &= \int \frac{x^3 + x^2 - 3x - 3}{3x^2} dx = \\ &= \int \left(\frac{1}{3} x + \frac{1}{3} - \frac{1}{x} - \frac{1}{x^2} \right) dx = \frac{1}{3} \int x dx + \frac{1}{3} \int dx - \int \frac{dx}{x} - \\ &\quad - \int x^{-2} dx = \frac{1}{6} x^2 + \frac{1}{3} x - \ln x + \frac{1}{x} + C. \end{aligned}$$

$$\begin{aligned} \int \frac{(x - \sqrt{x})(1 + \sqrt{x})}{\sqrt[3]{x}} dx &= \int \frac{x\sqrt{x} - \sqrt{x}}{\sqrt[3]{x}} dx = \\ &= \int x^{\frac{7}{6}} dx - \int x^{\frac{1}{6}} dx = \frac{6}{13} x^{\frac{13}{6}} - \frac{6}{7} x^{\frac{7}{6}} + C \end{aligned}$$