

$$\int \frac{dx}{x^2 - a^2}$$

$$S(dx/(x^2-a^2))=S(dx/(x-a)(x+a))=S(dx/(x-a)(x+a))$$

$$(0 \cdot x + 1)/(x-a)(x+a) = 1/(x-a)(x+a) = A/(x-a) + B/(x+a) = (A(x+a) + B(x-a))/(x+a)(x-a) = (Ax + Aa + Bx - Ba)/(x+a)(x-a) = (x(A+B) + a(A-B))/(x+a)(x-a)$$

$$(A+B)=0$$
$$a(A-B)=1$$

$$A=-B$$
$$a(-2B)=1$$
$$B=-1/2a$$
$$A=1/2a$$

$$1/(x-a)(x+a) = A/(x-a) + B/(x+a) = 1/[2a(x-a)] - 1/[2a(x+a)] = 1/2a(1/(x-a) - 1/(x+a))$$
$$S(1/2a(1/(x-a) - 1/(x+a)))dx = (1/2a)S((1/(x-a))dx) - (1/2a)S((1/(x+a))dx) =$$
$$= (1/2a)S((1/(x-a))d(x-a)) - (1/2a)S((1/(x+a))d(x+a)) =$$
$$= (1/2a) \cdot \ln|x-a| - (1/2a) \cdot \ln|x+a| + C = (1/2a)[\ln|x-a| - \ln|x+a|] + C = (1/2a)\ln|(x-a)/(x+a)| + C$$