

$$\int \sin mx \cos nx \, dx$$

$$\int \cos mx \cos nx \, dx$$

$$\int \sin mx \sin nx \, dx$$

$$\begin{aligned} \int \sin mx \cos nx \, dx &= \int \frac{1}{2} (\sin(m+n)x + \sin(m-n)x) \, dx = \\ &= \frac{1}{2} \int \sin(m+n)x \, dx + \frac{1}{2} \int \sin(m-n)x \, dx = \\ &= \frac{1}{2} \int \sin t \, dt / (m+n) + \frac{1}{2} \int \sin u \, du / (m-n) = \frac{1}{2(m+n)} (-\cos t) + \\ &+ \frac{1}{2(m-n)} (-\cos u) = \frac{1}{2(m+n)} (-\cos(m+n)x) + \frac{1}{2(m-n)} (-\cos(m-n)x) + C \\ t &= (m+n)x \\ u &= (m-n)x \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \int \sin(m+n)x \, dx &= \frac{1}{2(m+n)} \int \sin t \, dt = \\ &= -\frac{1}{2(m+n)} \cos t = -\frac{1}{2(m+n)} \cos(m+n)x \end{aligned}$$

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$$\sin mx \cos nx = \frac{1}{2} [\sin(m+n)x + \sin(m-n)x],$$

$$\cos mx \cos nx = \frac{1}{2} [\cos(m+n)x + \cos(m-n)x],$$

$$\sin mx \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x].$$