$$\int \frac{\sin 2nx}{\sin x}\,dx$$

S(sin2nx)dx/sinx=
2S(sin2nx)dx/2sinx=
2S(cosx+cos3x+cos5x+...+cos(2n-1)x)dx=
2(sinx+sin3x/3+sin5x/5+...+sin(2n-1)x/(2n-1))+C

$$\sin mx \cos nx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x],$$

$$\cos mx \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x],$$

$$\sin mx \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x].$$

$$\int \frac{\sin(2n+1)x}{\sin x}\,dx$$

S(sin2n+1)xdx/sinx=
2S(sin2n+1)xdx/2sinx=
2S(½+cos2x+cos4x+cos6x+...+cos(2n)x)dx=
x+sin2x+sin4x/2+sin6x/3+...+sin(2nx)/n+C

cosx+cos3x+cos5x+...+cos(2n-1)x=A |*sinx
cosx*sinx+cos3x*sinx+cos5x*sinx+...+cos(2n-1)x*sinx=A*sinx
1/2sin2x+½(sin(4x)-sin(2x))+½(cos(6x)-sin(4x))+...+½(sin(2nx)-sin((2n-2)x)=A*sinx
½[sin2x+sin(4x)-sin(2x)+sin(6x)-sin(4x)+...+sin(2nx)-sin((2n-2)x]=A*sinx
½[sin(2nx)]=A*sinx
A=sin2nx/2sinx

cosx+cos3x+cos5x+...+cos(2n-1)x.+cos(2n+1)x=A |*sinx
cosx*sinx+cos3x*sinx+cos5x*sinx+...+cos(2n-1)x*sinx+cos(2n+1)x*sinx=A*sinx
1/2sin2x+½(sin(4x)-sin(2x))+½(cos(6x)-sin(4x))+...+½(sin(2nx)-sin((2n-2)x)+½[sin(x-(2n+1)x)+sin((2n+1)x+x)]=A*sinx
1/2sin2x+½(sin(4x)-sin(2x))+½(cos(6x)-sin(4x))+...+½(sin(2nx)-sin((2n-2)x)+½[-sin(2nx)+sin(2nx+2x)]=A*sinx
½* sin(2n+2)x= Asinx

cos2x+cos4x+cos6x+...+cos(2n)x=A |*sinx
cos2x*sinx+cos4x*sinx+cos6x*sinx+...+cos(2n)x*sinx=A*sinx
½{[sin3x-sinx]+[sin5x-sin3x]+[sin7x-sin5x]+….+[sin(x+2nx)-sin(2nx-x)]}=A*sinx
-½sinx+½sin(x+2nx)=A*sinx
A=sin((1+2n)x)/2sinx-½

sin((1+2n)x)/2sinx=A+1/2

sinx+sin3x+sin5x+...+sin(2n-1)x.+sin(2n+1)x=A |*cosx
sinx*cosx+sin3x*cosx+sin5x*cosx+...+sin(2n-1)x*cosx=A*cosx
½[sin(2nx)+sin(2nx-2x)]=A*cosx