

$$(a) \int \frac{dx}{\sin x} \quad (b) \int \frac{dx}{\cos x}$$

$$\begin{aligned} S(dx/\sin x) &= S(dx/2\sin(x/2)\cos(x/2))= \\ S(dx/\cos^2(x/2) / 2\sin(x/2)\cos(x/2)/\cos^2(x/2)) &= \\ = S(2d(x/2)/\cos^2(x/2) / 2\sin(x/2)\cos(x/2)/\cos^2(x/2)) &= \\ = S(d(x/2)/\cos^2(x/2) / \sin(x/2)\cos(x/2)/\cos^2(x/2)) &= \\ S(d(\operatorname{tg}(x/2)) / \sin(x/2)/\cos(x/2)) &= \\ S(d(\operatorname{tg}(x/2)) / \operatorname{tg}(x/2)) &= [t=\operatorname{tg}(x/2)] = S(d(t) / t) = \ln|t| + C = \ln|\operatorname{tg}x/2| + C \end{aligned}$$

$$(1+\operatorname{tg}(x/2))/(1-\operatorname{tg}(x/2))$$

$$\begin{aligned} S(1/(1-t))dt &= -S(1/(1-t))d(-t) \\ &= -S(1/(1-t))d(1-t) = [1-t=y] = \\ &= -S(1/y)dy = -\ln|y| = -\ln|1-t| \end{aligned}$$

$$\begin{aligned} S(dx/\cos x) &= S(dx/(\cos^2(x/2)-\sin^2(x/2)))= \\ S(dx/\cos^2(x/2)/(\cos^2(x/2)-\sin^2(x/2))/\cos^2(x/2)) &= \\ S(2d(\operatorname{tg}(x/2))/(1-\operatorname{tg}^2(x/2))) &= [t=\operatorname{tg}(x/2)] = 2S(d(t)/(1-t^2)) \end{aligned}$$

$$\begin{aligned} 1/(1-t^2) &= A/(1-t) + B/(1+t) = (A(1+t) + B(1-t))/(1-t)(1+t) = \\ &= (A+At + B-Bt)/(1-t)(1+t) = ((A+B)+t(A-B))/(1-t)(1+t) \end{aligned}$$

$$\begin{aligned} (A-B) &= 0 \\ (A+B) &= 1 \\ A &= B \\ 2B &= 1 \\ B &= A = 1/2 \end{aligned}$$

$$\begin{aligned} 2S(1/2(1-t) + 1/2(1+t)) &= S(1/(1-t) + 1/(1+t))dt = \\ &= S(1/(1-t))dt + S(1/(1+t))dt = \\ &= -\ln|1-t| + \ln|1+t| + C = -\ln|1-\operatorname{tg}(x/2)| + \ln|1+\operatorname{tg}(x/2)| + C = \\ &= \ln|1+\operatorname{tg}(x/2)|/|1-\operatorname{tg}(x/2)| + C = \ln|(1+\operatorname{tg}(x/2))/(1-\operatorname{tg}(x/2))| + C \end{aligned}$$

$$\begin{aligned} S(dx/\cos x) &= S(dx/\sin(x+P/2))= \\ &= S(d(x+P/2)/\sin(x+P/2))= \\ &= [(x+P/2)=u] = Sdu/\sin(u) = \\ &= \ln|\operatorname{tg}u/2| + C = \ln|\operatorname{tg}(x+P/2)/2| + C = \\ &= \ln|\operatorname{tg}(x/2+P/4)| + C \end{aligned}$$

$$\begin{aligned} \sin(x+P/2) &= \sin x \cos P/2 + \cos x \sin P/2 = \cos x \end{aligned}$$

$$\begin{aligned} \operatorname{tg}(x/2+P/4) &= \sin(x/2+P/4)/ \\ &= \sin(x/2)\cos P/4 + \cos x/2\sin P/4 / \\ &= [\sin x/2\cos P/4 + \cos x/2\sin P/4] / \\ &= \sqrt{2}/2[\sin x/2 + \cos x/2] / \\ &= \sqrt{2}/2[\cos x/2 - \sin x/2] = \\ &= [\sin x/2 + \cos x/2] / [\cos x/2 - \sin x/2] = \\ &= [\sin x/2 + \cos x/2] / \cos x/2 = \\ &= [\operatorname{tg}x/2 + 1] / [1 - \operatorname{tg}x/2] \end{aligned}$$