

$$\int \frac{dx}{(x^2 + a^2)^2}$$

$$S(dx/(x^2+a^2)^2)=S(dx/(x^2+a^2)^2)$$

$$x=a \cdot \operatorname{tg}(t) \Rightarrow \operatorname{tgt}=x/a \Rightarrow t=\operatorname{arctg}(x/a)$$

$$dx=a \cdot d(\operatorname{tgt})=a \cdot dt/\cos^2 t$$

$$x^2+a^2=a^2 \cdot \operatorname{tg}^2(t) + a^2=a^2(\operatorname{tg}^2(t)+1)=a^2/\cos^2 t$$

$$S(a \cdot dt/\cos^2 t / (a^2/\cos^2 t)^2)=S(a \cdot dt \cdot \cos^2 t / (a^4))=$$

$$1/a^3 S(dt \cdot \cos^2 t)=1/a^3 (t/2 + \sin 2t/4) + C =$$

$$=1/a^3 (\operatorname{arctg}(x/a)/2 + \sin 2 \operatorname{arctg}(x/a)/4) + C = 1/a^3 (\operatorname{arctg}(x/a)/2 - 2ax/4a^3(a^2+x^2)) + C =$$

$$=1/a^3 (\operatorname{arctg}(x/a)/2 - x/[2a^2(a^2+x^2)]) + C$$

$$S(\cos^2(mx))dx=S((1+\cos 2mx)/2)dx=S(1/2)dx+S(\cos 2mx/2)dx=$$

$$=x/2 + 1/2 S(\cos 2mx)dx=x/2 + 1/2 S(\cos 2mx/2m)d(2mx)=$$

$$=x/2 + \sin 2mx/4m + C$$

$$\sin 2 \operatorname{arctg}(x/a) = \sin 2u = 2 \sin u \cos u = +2a/\sqrt{x^2+a^2} \cdot x/\sqrt{a^2+x^2} = +2ax/(a^2+x^2)$$

$$\operatorname{arctg}(x/a) = u \in (-\pi/2; \pi/2) \Rightarrow \cos u > 0 \quad x/a = \operatorname{tgu}$$

$$2u \in (-\pi; \pi)$$

$$1 + \operatorname{tg}^2 u = 1/\cos^2 u$$

$$\cos^2 u = 1/(1 + \operatorname{tg}^2 u)$$

$$\cos u = \pm 1/\sqrt{1 + \operatorname{tg}^2 u} = +1/\sqrt{1 + \operatorname{tg}^2 u} = +1/\sqrt{1 + (x/a)^2} = a/\sqrt{x^2 + a^2}$$

$$\sin u = \pm \sqrt{1 - \cos^2 u} = \pm \sqrt{1 - 1/(1 + (x/a)^2)} = \pm \sqrt{[(1 + (x/a)^2) - 1]/(1 + (x/a)^2)} =$$

$$= \pm \sqrt{(x/a)^2 / (1 + (x/a)^2)} = \pm \sqrt{x^2 / (a^2 + x^2)} = \pm x/\sqrt{a^2 + x^2}$$

