

$$(a) \int \frac{dx}{(x^2 + a^2)^{3/2}}, (6) \int \frac{dx}{(x^2 - a^2)^{3/2}},$$



$$\int \frac{dx}{(a^2 - x^2)^{3/2}}$$

$$\text{ch}^2(t) - \text{sh}^2(t) = 1$$

$$S(dx/(x^2 - a^2)^{3/2}) = [x = a \cdot \text{sect} = a/\text{cost}]$$

$$dx = d(a/\text{cost}) = ad(1/\text{cost}) = a \cdot \text{sint} \cdot dt / \cos^2 t = a \cdot \text{tg} t \cdot dt / \cos t$$

$$(1/\text{cost})' = (\text{cost}^{-1})' = -1 \cdot \text{cos}^{-2} t = -\text{tg} t / \cos^2 t$$

$$x^2 - a^2 = (a/\text{cost})^2 - a^2 = a^2(1/\cos^2 t - 1) = a^2(1 - \cos^2 t) / \cos^2 t = a^2(\sin^2 t) / \cos^2 t = a^2 \cdot \text{tg}^2 t$$

$$S(a \cdot \text{tg} t \cdot dt / \cos t / (a^2 \cdot \text{tg}^2 t)^{3/2}) = S(dt / \text{cost} \cdot a^2 \cdot \text{tg}^2 t) =$$

$$= 1/a^2 S(\text{cost} \cdot dt / \sin^2 t) = 1/a^2 S(\text{dsint} / \sin^2 t) = [\text{sint} = y] =$$

$$= 1/a^2 S(dy / y^2) = 1/a^2 (-y^{-1}) + C = -1/a^2 \cdot 1/(\text{sint}) + C = -1/a^2 \cdot 1/(-V(1 - (a/x)^2)) + C$$

$$\text{cost} = a/x$$

$$\sin^2 t = 1 - \cos^2 t = 1 - (a/x)^2$$

$$\text{sint} = +V(1 - (a/x)^2)$$

$$S(dx/(x^2 + a^2)^{3/2}) =$$

$$x = a \cdot \text{sht}$$

$$dx = d(a \cdot \text{sht}) = a \cdot \text{cht} \cdot dt$$

$$S(dx/(x^2 + a^2)^{3/2}) = 1/a^2 S(\text{cht} \cdot dt / (\text{sht}^2 + 1)^{3/2}) =$$

$$1/a^2 S(\text{cht} \cdot dt / (\text{sh}^2 t + \text{ch}^2 t - \text{sh}^2 t)^{3/2}) =$$

$$1/a^2 S(\text{cht} \cdot dt / (\text{ch}^2 t)^{3/2}) = 1/a^2 S(\text{cht} \cdot dt / (\text{ch}^3 t)) =$$

$$1/a^2 S(dt / (\text{ch}^2 t)) = 1/a^2 \cdot \text{tcht} + C = 1/a^2 \cdot x/a / V(1 + x^2/a^2) + C$$

$$\text{sht} = x/a$$

$$\text{cht} = V(1 + \text{sh}^2 t) = V(1 + x^2/a^2)$$

$$(\text{cht} > 0 \text{ - всегда})$$

$$S(dx/(a^2 - x^2)^{3/2}) = [x = a \cdot \text{sint}]$$

$$dx = d(a \cdot \text{sint}) = a \cdot \text{cost} \cdot dt$$

$$S(a \cdot \text{cost} \cdot dt / (a^2 - (a \cdot \text{sint})^2)^{3/2}) =$$

$$= 1/a^2 S(\text{cost} \cdot dt / (1 - (\text{sint})^2)^{3/2}) =$$

$$= 1/a^2 S(\text{cost} \cdot dt / (\cos^2 t)^{3/2}) =$$

$$= 1/a^2 S(\text{cost} \cdot dt / (\cos^3 t)) =$$

$$= 1/a^2 S(dt / (\cos^2 t)) = 1/a^2 \cdot \text{tgt} + C =$$

$$= 1/a^2 \cdot x / (+aV(1 - (x/a)^2)) + C$$

$$\text{sint} = x/a$$

$$\text{cost} = +V(1 - (x/a)^2)$$