$$\int \frac{dx}{x\sqrt{a^2-x^2}}$$

log_a(b)^n=n*log_a(b)

окончание 1 способа:
-1/a*1/2ln|(a+V(a^2-x^2))/(a-V(a^2-x^2))|+C=
=1/a*1/2ln|(a-V(a^2-x^2))/(a+V(a^2-x^2))|+C=
=1/a*1/2ln|(a-V(a^2-x^2)^2)/(a-V(a^2-x^2)(a+V(a^2-x^2))|+C=
=1/a*1/2ln|(a-V(a^2-x^2)^2)/(a^2-(a^2-x^2))|+C=
=1/a*1/2ln|(a-V(a^2-x^2)^2)/(x^2)|+C=
=1/a*1/2ln|{(a-V(a^2-x^2))/(x)}^2|+C=
=1/a*2/2ln|(a-V(a^2-x^2))/(x)|+C=
=1/a*ln|(a-V(a^2-x^2))/(x)|+C

2 СПОСОБ 1/aS(dt/sint)
S(dt/sint)=S(dt/2sint/2*cost/2)=S(dt/cos^(t/2) / 2sint/2*cost/2 / cos^2 (t/2))=S(2*d(t/2)/cos^(t/2) / 2sint/2/cos (t/2) =
=S(d(tg(t/2) / tg(t/2) = [tg(t/2)=h] = Sdh / h =
=ln|h|+ C= ln|tg(t/2)|+ C
sint=x/a
tg(t/2)=y
sint=2sint/2cost/2 / 1 = 2sint/2cost/2 / (sin^2 t/2 + cos^2 t/2) =
= 2sint/2cost/2 / cos^2 t/2 / (sin^2 t/2 + cos^2 t/2) / cos^2 t/2=
= 2tg t/2 / (tg^2 t/2 + 1)
sint = 2y/(1+y^2)
sint(1+y^2)=2y
sint+sinty^2=2y
sinty^2-2y+sint=0
D=4-4sin^2t
VD=V(4-4sin^2t)=2V(1-sin^2t)=2V(1-(1-cos^2t))=2V(cos^2t)=2cost
y=(2+-2cost)/2sint=(1+-cost)/sint=a(1+-V(1-x^2/a^2))/x=
=a((a+-V(a^2-x^2)/a)/x=(a+-V(a^2-x^2))/x

S=1/a ln|(a+-V(a^2-x^2))/x| + C

1 СПОСОБ
S(dx/xV(a^2-x^2))=[x=asint]
dx=acostdt
S(acostdt/asintV(a^2-a^2sin^2t))
1/aS(costdt/sintV(sin^2t+cos^2t-sin^2t))=
=1/aS(dt/sint)=1/aS(dt/sint)

1/aS(sintdt/sin^2t)=1/aS(sintdt/(1-cos^2t))=
=1/aS(sintdt/(1-cos^2t))=1/aS(-d(cost)/(1-cos^2t))=
=[cost=y]=-1/aS(d(y)/(1-y^2))=-1/aS((A+B)dy/(1-y^2))=

(A+B)/(1-y^2)=A/(1-y)+B/(1+y)=
=(A(1+y)+B(1-y))/(1-y)(1+y)=(A+Ay+B-By)/(1-y)(1+y)=
(A+B+y(A-B))/(1-y)(1+y)=

A-B=0
A+B=1

A=B
B=½

A/(1-y)+B/(1+y)=1/2 (1-y)+1/2 (1+y)
S(1/2 (1-y)+1/2 (1+y))dy=1/2S(1/(1-y)+1/(1+y))dy=
1/2S(1/(1-y))*(-1)d(-y+1)+1/2S(1/(1+y))d(y+1)=[1-y=z;
1+y=c]=1/2S(1/z)*(-1)dz+1/2S(1/c)dc=1/2ln|c|-1/2ln|z|+C=
1/2ln|c/z|+C=1/2ln|(1+y)/(1-y)|+C=1/2ln|(1+cost)/(1-cost)|+C=
1/2ln|(1+V(1-x^2/a^2))/(1-V(1-x^2/a^2))|+C=
1/2ln|(1+V(a^2-x^2)/a)/(1-V(a^2-x^2)/a)|+C=
1/2ln|(a+V(a^2-x^2))/(a-V(a^2-x^2))|+C=

sint=x/a
cost=+V(1-x^2/a^2)

универсальная тригонометрическая подстановка
tgx/2 = z

sinx=2sinx/2cosx/2 / 1 = 2sinx/2cosx/2 / (sin^2 x/2 + cos^2 x/2) =
= 2sinx/2cosx/2 / cos^2x/2 / (sin^2 x/2 + cos^2 x/2) / cos^2 x/2=
= 2tg x/2 / (tg^2 x/2 + 1)=2z / (z^2 + 1)

cosx=[cos^2(x/2)-sin^2(x/2)] / 1 = [cos^2(x/2)-sin^2(x/2)] / (sin^2(x/2)+cos^2(x/2))= [cos^2(x/2)-sin^2(x/2)]/cos^2x/2 / (sin^2(x/2)+cos^2(x/2))/cos^2x/2= [1-tg^2(x/2)] / (tg^2(x/2)+1)= [1-z^2]/[1+z^2]

tgx=sinx/cosx=2z / (z^2 + 1)/[1-z^2]/[1+z^2]=
=2z /[1-z^2]

ctgx=[1-z^2]/2z

cos2x + sin^2 x - 5= 0
z^5-3z+6=0