

Решите неравенство

$$\frac{81^x + 2 \cdot 25^{x \log_5 3} - 5}{(4x-1)^2} \geq 0.$$

$$(81^x + 2 \cdot 5^{(2x \log_5 3)} - 5) / (4x-1)^2 \geq 0$$

$$(81^x + 2 \cdot 3^{2x} - 5) / (4x-1)^2 \geq 0$$

$$(81^x + 2 \cdot 9^x - 5) / (4x-1)^2 \geq 0$$

$$81^x + 2 \cdot 9^x - 5 \geq 0 \quad x \neq 1/4$$

$$9^{2x} + 2 \cdot 9^x - 5 \geq 0$$

$$t = 9^x$$

$$t^2 + 2t - 5 \geq 0$$

$$t = -1 \pm \sqrt{6}$$

$$t \in (-\infty; -1 - \sqrt{6}] \cup [-1 + \sqrt{6}; +\infty)$$

$$9^x \in (-\infty; -1 - \sqrt{6}] \cup [-1 + \sqrt{6}; +\infty)$$

$$9^x \leq -1 - \sqrt{6} \text{ (no sol)}$$

$$9^x \geq -1 + \sqrt{6}$$

$$\log_9(9^x) \geq \log_9(-1 + \sqrt{6})$$

$$x \geq \log_3^2(-1 + \sqrt{6})$$

$$x \geq \frac{1}{2} \cdot \log_3(-1 + \sqrt{6})$$

$$x \in [\frac{1}{2} \cdot \log_3(-1 + \sqrt{6}); \frac{1}{4}) \cup (\frac{1}{4}; +\infty)$$

$$\text{Ответ: } [\frac{1}{2} \cdot \log_3(-1 + \sqrt{6}); \frac{1}{4}) \cup (\frac{1}{4}; +\infty)$$