

Найдите все значения a , при каждом из которых система

$$\begin{cases} x = \sin t \\ y = \cos t \end{cases} \quad [12; 23]$$

$$\begin{cases} 5 \cdot 2^{|x|} + 6|x| + 7 = 5y + 6x^2 + 4a, \\ x^2 + y^2 = 1 \end{cases} \quad [-5+4a; 169/24+4a]$$

имеет единственное решение.

$$5 \cdot 2^{|\sin t|} + 6|\sin t| + 7 = 5 \cos t + 6 \sin^2 t + 4a$$

$$5 \cdot 2^{|x|} + 6|x| + 7 = 5y + 6x^2 + 4a$$

$$5y + 6x^2 + 4a > 0$$

$$x^2 = 1 - y^2$$

$$5 \cdot 2^{|x|} + 6|x| + 7 = 5y + 6(1 - y^2) + 4a$$

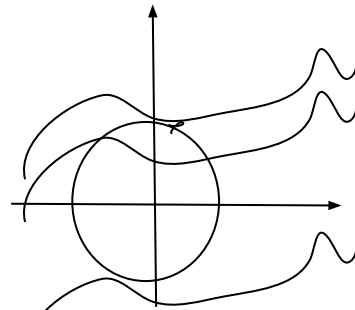
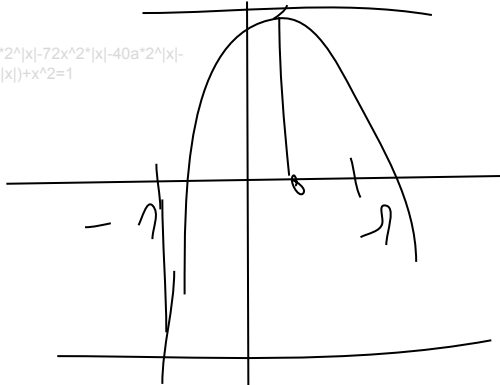
$$5 \cdot 2^{|x|} + 6|x| + 7 = 5y + 6 - 6y^2 + 4a$$

$$5 \cdot 2^{|x|} + 6|x| + 1 = 5y - 6y^2 + 4a$$

$$(5 \cdot 2^{|x|} + 6|x| + 7 - 6x^2 - 4a) / 5 = y$$

$$(5 \cdot 2^{|x|} + 6|x| + 7 - 6x^2 - 4a)^2 / 25 + x^2 = 1$$

$$25 \cdot 4^{|x|} + 36x^2 + 49 + 36x^4 + 16a^2 + 84|x| - 60x^2 \cdot 2^{|x|} - 72x^2 \cdot |x| - 40a^2 \cdot |x| - 48a \cdot |x| - 56a + 48ax^2 + 30 \cdot |x|^2 \cdot (1 + |x|) + 35 \cdot 2^{|x|} (1 + |x|) + x^2 = 1$$



$$\begin{aligned} -5 + 4a &= 23 \\ a &= (23 + 5) / 4 = 7 \end{aligned}$$

$$\begin{aligned} 1 &= 2^0 \leq 2^{|\sin t|} \leq 2^1 \leq 2 \\ 5 &\leq 5 \cdot 2^{|\sin t|} \leq 10 \\ 0 \cdot 6 &\leq 6 \cdot |\sin t| \leq 6 \end{aligned}$$

$$\begin{aligned} 169/24 + 4a &= 12 \\ 4a &= (288 - 169) / 24 = 119/24 \\ a &= 119/96 \end{aligned}$$

$$\begin{aligned} y(t) &= 5 \cos t + 6 \sin^2 t + 4a \\ y(t) &= 5 \cos t + 6 - 6 \cos^2 t + 4a \\ \cos t &= z, \quad |z| \leq 1 \end{aligned}$$

$$\begin{aligned} y(z) &= -6z^2 + 5z + 6 + 4a \\ z_B &= 5/12 \quad \text{max} \end{aligned}$$

$$\begin{aligned} -1 &\quad \text{min} \\ y(5/12) &= -6(5/12)^2 + 25/12 + 6 + 4a \\ &= -25/24 + 25/12 + 6 + 4a = \\ &= 169/24 + 4a \\ y(-1) &= -6 - 5 + 6 + 4a = -5 + 4a \end{aligned}$$