

Найдите все значения параметра a , при каждом из которых система уравнений

$$\begin{cases} 2x^2 + 2y^2 = 5xy, \\ (x-a)^2 + (y-a)^2 = 5a^4 \end{cases}$$

имеет ровно два решения.

$$2x^2 + 2y^2 = 5xy$$

$$2\left(x - \frac{5y}{4}\right)^2 - \frac{9y^2}{8} = 0$$

$$2\left(x - \frac{5y}{4}\right)^2 - \frac{9y^2}{8} = 0$$

$$2\left(x^2 - \frac{5xy}{2} + \frac{25y^2}{16}\right) - \frac{9y^2}{8} = 0$$

$$2x^2 - 5xy + \frac{25y^2}{8} - \frac{9y^2}{8} = 0$$

$$2x^2 - 5xy + \frac{16y^2}{8} = 0$$

$$2x^2 - 5xy + 2y^2 = 0$$

$$(\sqrt{2}x - \frac{5\sqrt{2}y}{4})^2 - \left(\frac{3y}{2\sqrt{2}}\right)^2 = 0$$

$$(\sqrt{2}x - \frac{5\sqrt{2}y}{4} - \frac{3y}{2\sqrt{2}})(\sqrt{2}x - \frac{5\sqrt{2}y}{4} + \frac{3y}{2\sqrt{2}}) = 0$$

$$(\sqrt{2}x - \frac{5\sqrt{2}y}{4} - \frac{3\sqrt{2}y}{4})(\sqrt{2}x - \frac{5\sqrt{2}y}{4} + \frac{3\sqrt{2}y}{4}) = 0$$

$$(\sqrt{2}x - \frac{8\sqrt{2}y}{4})(\sqrt{2}x - \frac{2\sqrt{2}y}{4}) = 0$$

$$(x - \frac{8y}{4})(x - \frac{2y}{4}) = 0$$

$$(x - 2y)(x - \frac{y}{2}) = 0$$

$$x = 2y \parallel x = \frac{y}{2}$$