

$$\begin{aligned} \int \frac{dx}{\sqrt{x(1+x^{1/3})}} &= \int \frac{6t^5 dt}{t^3(1+t^2)} = \int \frac{6t^2 dt}{(1+t^2)} = \\ &= 6 \int \frac{(t^2+1-1)}{(1+t^2)} dt = 6 \int \frac{(t^2+1)}{(1+t^2)} + \frac{(-1)}{(1+t^2)} dt = \\ &= 6t + \int \frac{-1}{(1+t^2)} dt = 6t - \arctan t + C = 6x^{1/6} - \arctan x^{1/6} + C \end{aligned}$$

$$x^{1/6} = t$$

$$x^{1/2} = t^3$$

$$x^{1/3} = t^2$$

$$x = t^6$$

$$dx = d(t^6) = 6t^5 dt$$