

$$\begin{aligned}
\int \frac{x+1}{\sqrt{x^2+1}} dx &= \int \frac{x}{\sqrt{x^2+1}} dx + \int \frac{1}{\sqrt{x^2+1}} dx = \\
&= \ln|x + \sqrt{x^2+1}| + \int \frac{x}{\sqrt{x^2+1}} dx + C = \\
&= \ln|x + \sqrt{x^2+1}| + \int \frac{1}{\sqrt{x^2+1}} d\left(\frac{x^2}{2}\right) + C = \\
&= \ln|x + \sqrt{x^2+1}| + \frac{1}{2} \int \frac{1}{\sqrt{x^2+1}} d(x^2+1) + C = \\
&= \ln|x + \sqrt{x^2+1}| + \frac{1}{2} \int \frac{1}{\sqrt{t}} dt + C = \\
&= \ln|x + \sqrt{x^2+1}| + \frac{1}{2} \int t^{-1/2} dt + C = \\
&= \ln|x + \sqrt{x^2+1}| + \frac{1}{2} \cdot 2t^{1/2} + C = \ln|x + \sqrt{x^2+1}| + \sqrt{x^2+1} + C
\end{aligned}$$

$x dx = d(x^2/2)$   
 $x^2 + 1 = t$