

$$(1+x)^n > 1+nx \text{ при } x \geq -1, n \in \mathbb{N}$$

$$n=1$$

$$(1+x)^1 > 1+x$$

Let k hold

$$n=k$$

$$(1+x)^k > 1+kx$$

than for k+1 we have to prove:

$$(1+x)^{k+1} > 1+x(k+1)$$

$$(1+x)^{k+1} = (1+x)^k (1+x)$$

$$(1+x)^k (1+x) > (1+kx)(k+1)$$

$$(1+kx)(k+1) > 1+x(k+1)$$

$$1+x(k+1) = 1+xk+x$$

$$(1+kx)(k+1) = 1+x+kx^2+kx$$

$$1+x+kx^2+kx > 1+xk+x$$

$$(1+x)^{k+1} = (1+x)^k (1+x) > (1+kx)(k+1) = 1+x+kx^2+kx > 1+xk+x = 1+x(k+1)$$

$$b_n = \left(1 - \frac{1}{n}\right)^n$$

возрастающая

$$b_n = \left(1 - \frac{1}{n}\right)^n$$

$$b_{n+1} = \left(1 - \frac{1}{n+1}\right)^{n+1}$$

b_{n+1}/b_n must be no less than 1

$$b_{n+1}/b_n = \frac{\left(\frac{n}{n+1}\right)^{n+1}}{\left(\frac{n-1}{n}\right)^n} = \frac{n^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{(n-1)^n} =$$

$$= \frac{n^{2n+1}}{[(n+1)^{n+1} (n-1)^n]} = \frac{n^{2n+1}}{[(n+1)^n (n+1)^n (n-1)^n]} =$$

$$= \frac{n^{2n+1}}{(n+1)^n \{(n+1)(n-1)\}^n} = \frac{n^{2n+1}}{(n+1)^n \{n^2-1\}^n} = \frac{n^{2n+1}}{(n+1)^n \{n^2-1\}^n} =$$

$$= \frac{n}{n+1} \cdot \frac{n^{2n}}{\{n^2-1\}^n} = \frac{n}{n+1} \cdot \left[\frac{n^2}{n^2-1}\right]^n = \frac{n}{n+1} \cdot \left[\frac{n^2+1-1}{n^2-1}\right]^n =$$

$$= \frac{n}{n+1} \cdot \left[1 + \frac{1}{n^2-1}\right]^n > \frac{n}{n+1} \cdot \left[1 + \frac{1}{n^2}\right]^n > \frac{n}{n+1} \cdot \left[1 + \frac{1}{n}\right] =$$

$$= \frac{n}{n+1} \cdot \left[1 + \frac{1}{n}\right] = \frac{n}{n+1} \cdot \frac{n+1}{n} = 1$$