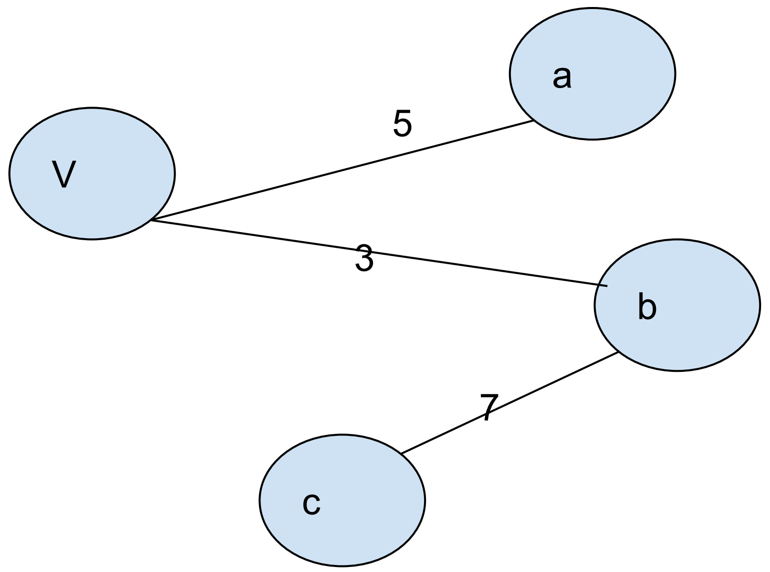
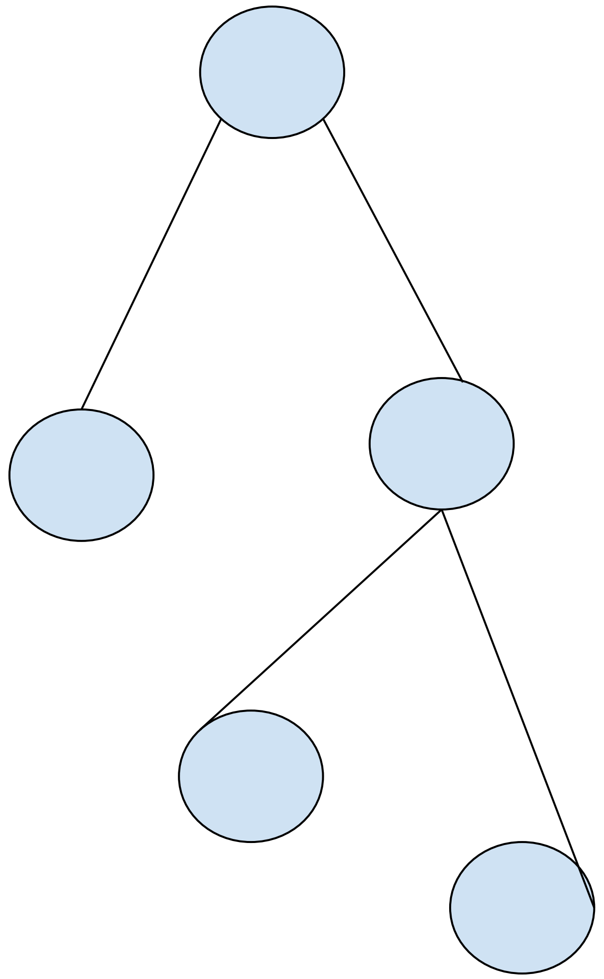
Exercise 9 [6 points] For a vertex v in a tree T we define the summed-distance $sumdist(v)$ of v to be the sum of all distances from v to any vertex in T. That is $sumdist_T(v) = \sum_{w \in V(T)} dist_T(v, w)$. For any tree T with $n \geq 2$ vertices the summed-distance of any vertex v is at most $n(n-1)/2$. Prove this claim using induction on trees.

Hint: Recall for induction on trees, to prove the statements for a tree T with $k+1$ vertices you can remove a leaf v to obtain a tree T' with k vertices on which you can apply the induction hypothesis. To prove the bound on the summed-distance for all of its $k+1$ vertices in T you may consider proving it separately for v and for the vertices that are also in T'.



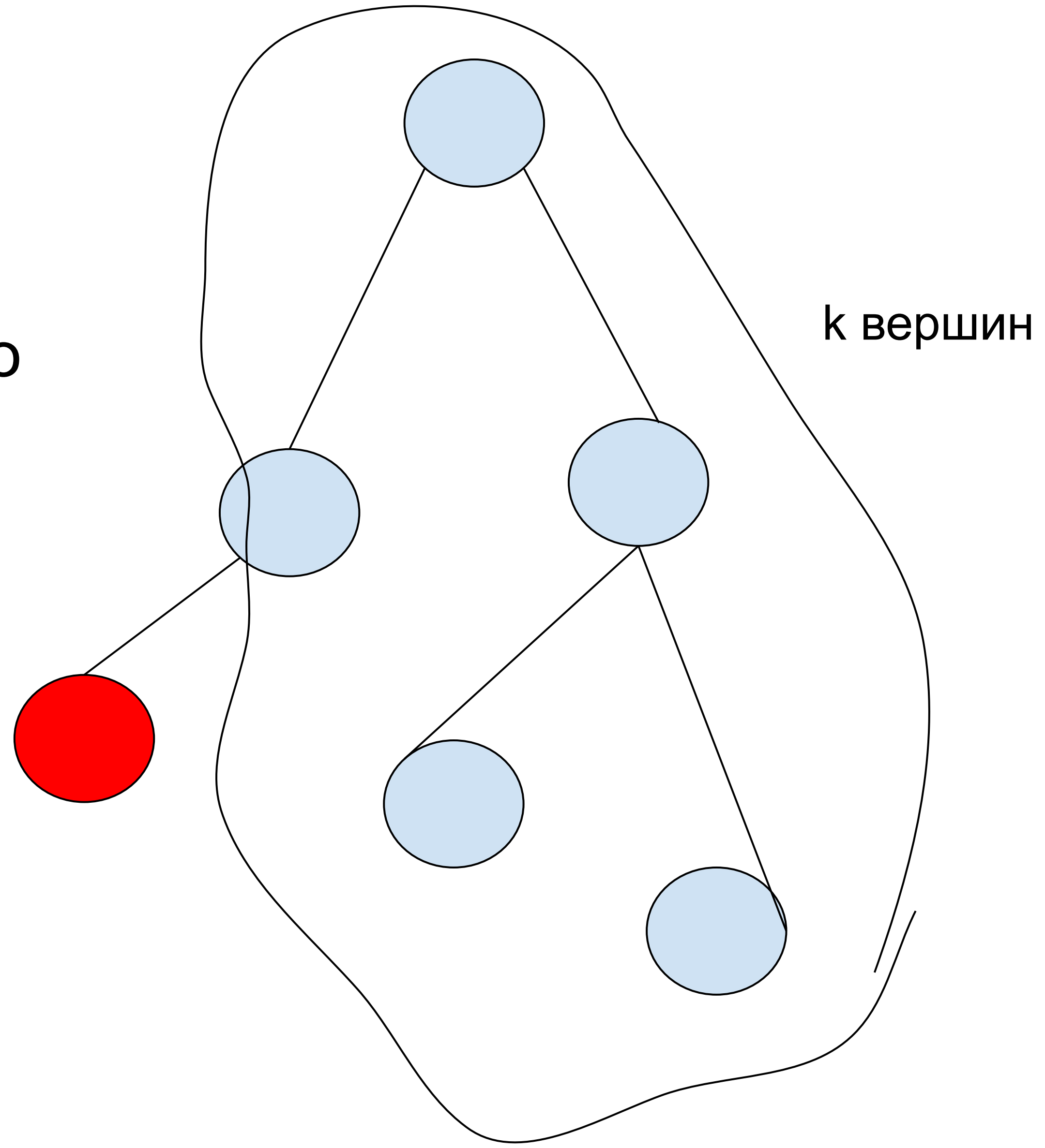
dist(a)=5
dist(b)=3
dist(c)=3+7=10
sumdist()=5+3+10



предположение
индукции

пусть для любой вершины в дереве из k вершин верно, что эта вершина имеет sum distance до других вершин не более, чем k(k-1)/2

Рассмотрим дерево из k+1 вершины. Докажем, что для него то же верно, что любая его вершина имеет sum distance до других вершин не более (k+1)k/2



k^2-k+2
D=1-4*2<0

k(k-1)/2 + 1
=(k(k-1)+2)/2=
=(k^2-k+2)/2 < (k+1)k/2

k(k-1)/2 + 1 ? (k+1)k/2
k(k-1) + 2 ? (k+1)k
k^2-k + 2 ? k^2+k
-k + 2 ? k