

$$S_n = a + aq + aq^2 + \dots + aq^{(n-2)} + aq^{(n-1)}$$

$$S_n = a + q(a + aq + \dots + aq^{(n-3)} + aq^{(n-2)}) = a + q(S_n - aq^{(n-1)})$$

$$S_n = x$$

$$x = a + q(x - aq^{(n-1)})$$

$$x = a + qx - aq^n$$

$$x - qx = a - aq^n$$

$$x(1 - q) = a - aq^n$$

$$x = a(1 - q^n) / (1 - q)$$

$$S_n = a(1 - q^n) / (1 - q)$$

$$|q| < 1 \Leftrightarrow -1 < q < 1$$

$$S_{\infty} = a(1 - 0) / (1 - q) = a / (1 - q)$$

