

$$P(x) / 1 \rightarrow \infty$$

$$P(x) / x \rightarrow 0$$

$$P(x) / x/\ln(x) \rightarrow 1$$

$$x/\ln(x) \sim P(x)$$

$$2x/\ln(2x) \sim P(2x)$$

$$\begin{aligned} P(2x) - P(x) &= 2x/\ln(2x) - x/\ln(x) = \\ &= x[2/\ln(2x) - 1/\ln(x)] > x[2/\ln(x) - 1/\ln(x)] = x[1/\ln(x)] = x/\ln(x) > 1 \end{aligned}$$