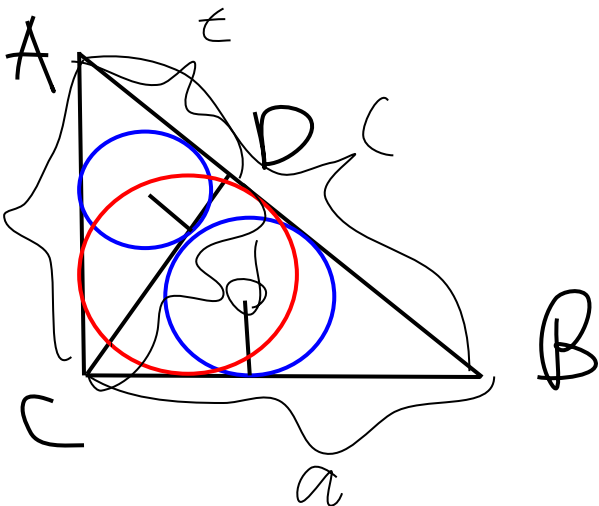


В прямоугольный $\triangle ABC$ (C - прямой) проведена высота CD . Радиусы окружностей, вписанных в ACD и BCD равны $r_1=6$ $r_2=8$. Найти радиус окружности, вписанной в $\triangle ABC$



$$r = S/p \quad S = r \cdot p$$

$$S_1 = 6 \cdot (b+t+h)/2 = 3 \cdot (b+t+h)$$

$$S_2 = 8 \cdot (a+c-t+h)/2 = 4 \cdot (a+c-t+h)$$

$$S(\text{общ}) = r \cdot (a+b+c)/2$$

$$S(\text{общ}) = S_1 + S_2$$

$$S_1 + S_2 = r \cdot (a+b+c)/2$$

$$3 \cdot (b+t+h) + 4 \cdot (a+c-t+h) = r \cdot (a+b+c)/2$$

$$6 \cdot (b+t+h) + 8 \cdot (a+c-t+h) = r \cdot (a+b+c)$$

$$6b + 6t + 6h + 8a + 8c - 8t + 8h = r \cdot (a+b+c)$$

$$8a + 6b - 2t + 8h = r \cdot (a+b+c)$$

$$r = 2 \cdot (4a + 3b - t + 4h) / (a+b+c)$$

$$ABC \sim ACD \quad b/t = a/d = c/b$$

$$ABC \sim BCD \quad a/(c-t) = b/d = c/a$$

$$ACD \sim BCD \quad a/b = (c-t)/d = d/t$$

$$S_1/S_2 = (6/8)^2 = (3/4)^2 = 9/16$$

$$S/S_1 = (r/6)^2 = r^2/36$$

$$S/S_2 = (r/8)^2 = r^2/64$$

$$S_1/S = 36/r^2$$

$$S_2/S = 64/r^2$$

$$S_1 + S_2 = S$$

$$S_1/S + S_2/S = 36/r^2 + 64/r^2$$

$$S/S = 100/r^2$$

$$r^2 = 100$$

$$r = 10$$

Ответ: 10