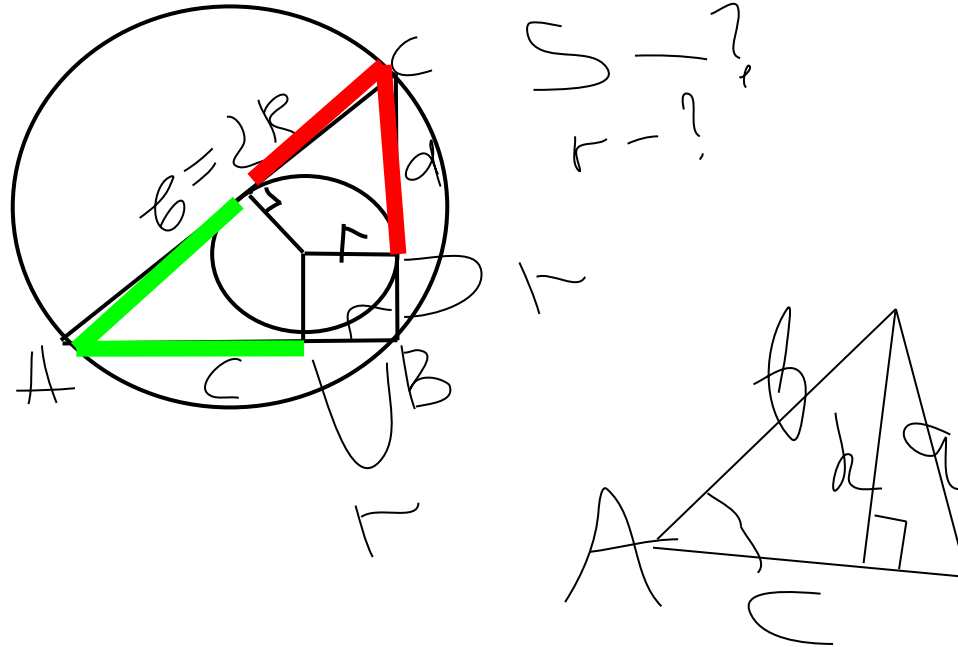


Радиусы описанной и вписанной окружности прямоугольного треугольника относятся как 5:2.
 Один из катетов равен "а". Найти площадь треугольника и радиус вписанной окружности.



S - ?
 r - ?

$$\text{зел} = 2R - \text{красн} = 2R - (a - r)$$

$$c = \text{зел} + r = 2R - (a - r) + r = 2R - a + 2r$$

напишем т пиф

$$(2R)^2 = a^2 + (2R - a + 2r)^2$$

$$\begin{aligned} (10x)^2 &= a^2 + (10x - a + 4x)^2 \\ 100x^2 &= a^2 + (14x - a)^2 \\ 100x^2 &= a^2 + 196x^2 - 28ax + a^2 \\ 100x^2 - a^2 - 196x^2 + 28ax - a^2 &= 0 \\ -96x^2 + 28ax - 2a^2 &= 0 \\ 14ax - a^2 - 48x^2 &= 0 \\ 48x^2 - 14ax + a^2 &= 0 \\ D/4 &= 49a^2 - 48a^2 = a^2 \\ x_1 &= (7a + a)/2 = 4a \\ x_2 &= (7a - a)/2 = 3a \end{aligned}$$

$$(2R)^2 = a^2 + (2R - a + 2r)^2$$

$$4R^2 = a^2 + (2R - a + 2r)^2$$

$$R = 5r/2$$

$$4(5r/2)^2 = a^2 + (2(5r/2) - a + 2r)^2$$

$$25r^2 = a^2 + (5r - a + 2r)^2$$

$$25r^2 = a^2 + (7r - a)^2$$

$$25r^2 = a^2 + 49r^2 - 14ra + a^2$$

$$12r^2 - 7ar + a^2 = 0$$

$$D = 49a^2 - 48a^2 = a^2$$

$$r_1 = (7a + a)/24 \quad r_2 = (7a - a)/24$$

$$r_1 = a/3 \quad r_2 = a/4$$

$$R_1 = 5a/6$$

$$R_2 = 5a/8$$

$$\begin{aligned} S_1 &= a \cdot V(4R^2 - a^2)/2 = \\ &= a \cdot V(4(5a/6)^2 - a^2)/2 = \\ &= a \cdot V(100a^2/36 - a^2)/2 = \\ &= a \cdot V(100a^2/36 - a^2)/2 = \\ &= a \cdot V(64a^2/36)/2 = 8a^2/12 = \\ &= 2a^2/3 \end{aligned}$$

$$\begin{aligned} S_2 &= a \cdot V(4(5a/8)^2 - a^2)/2 = \\ &= a \cdot V(100a^2/64 - a^2)/2 = \\ &= a \cdot V(100a^2 - 64a^2)/64)/2 = \\ &= a \cdot V(36a^2)/64)/2 = \\ &= 6a^2/16 = 3a^2/8 \end{aligned}$$

$$R/r = 5/2$$

$$S = \sin A \cdot AC \cdot AB / 2$$

$$S = \sin A \cdot b \cdot c / 2$$

$$\sin A = h/b$$

$$h = \sin A \cdot b$$

$$S = hc/2$$

$$S = \sin A \cdot b \cdot c / 2$$

$$a/\sin A = b/\sin B = c/\sin C = 2R$$

$$\sin A = a/2R$$

$$S = abc/(4R)$$

$$S = r(a+b+c)/2$$

$$b = 2R$$

$$c = V(4R^2 - a^2)$$

$$S = a \cdot V(4R^2 - a^2)/2 = r(a + 2R + V(4R^2 - a^2))/2$$

$$S = 2x(a + 10x + V(100x^2 - a^2))/2$$

$$2x(a + 10x + V(100x^2 - a^2))/2 = a \cdot V(100x^2 - a^2)/2$$

$$2xa + 20x^2 + 2x \cdot V(100x^2 - a^2) - a \cdot V(100x^2 - a^2) = 0$$

$$2xa + 20x^2 + (2x - a)V(100x^2 - a^2) = 0$$

$$2xa + 20x^2 = (a - 2x)V(100x^2 - a^2) \quad |^2$$

$$(2xa + 20x^2)^2 = (a - 2x)^2(100x^2 - a^2)$$

$$4x^2a^2 + 80x^3a + 400x^4 = (a^2 - 4xa + 4x^2) \cdot$$

$$\cdot (100x^2 - a^2)$$

$$4x^2a^2 + 80x^3a + 400x^4 = 100x^2a^2 - 400x^3a +$$

$$+ 400x^4 - a^4 + 4xa^3 - 4a^2x^2$$

$$4x^2a^2 + 80x^3a - 100x^2a^2 + 400x^3a + a^4 - 4xa^3 +$$

$$+ 4a^2x^2 = 0$$

$$480ax^3 - 92a^2x^2 - 4xa^3 + a^4 = 0 \quad | :a$$

$$480x^3 - 92ax^2 - 4a^2x + a^3 = 0$$