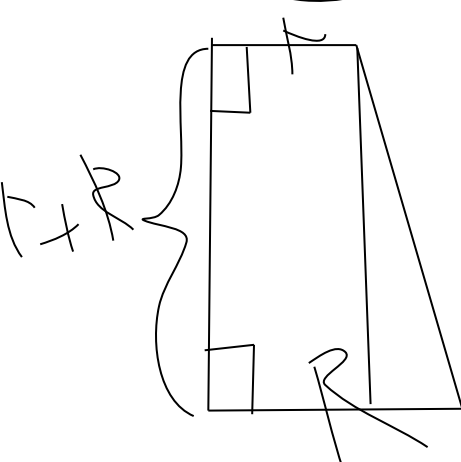
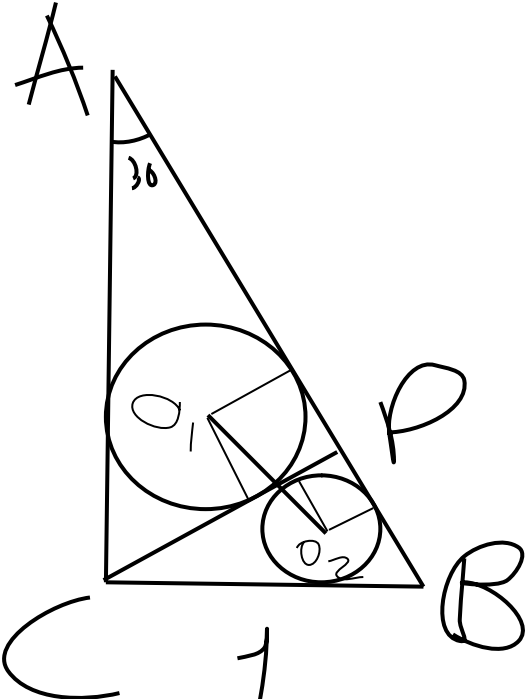


В прямоугольный треугольник ABC с острым углом 30 градусов проведена высота CD из вершины прямого угла C. Найти расстояние между центрами окружностей, вписанных в трACD и трBCD, если меньший катет ABC равен 1.



$$\begin{aligned} (R-r)^2 + (R+r)^2 &= \\ &= O_1O_2^2 \\ 2R^2 + 2r^2 &= \\ &= O_1O_2^2 \\ O_1O_2 &= \\ &= \sqrt{2R^2 + 2r^2} \end{aligned}$$

$$\begin{aligned} AB &= 2 \\ AC^2 &= 4 - 1 \\ AC^2 &= 3 \\ AC &= \sqrt{3} \\ CP &= \sqrt{3}/2 \\ BP^2 &= 1 - 3/4 = 1/4 \\ BP &= 1/2 \\ AP^2 &= 3 - 3/4 = 9/4 \\ AP &= 3/2 \\ S &= R(AC + CP + AP)/2 = R(\sqrt{3} + \sqrt{3}/2 + 3/2)/2 = \\ &= R(3\sqrt{3} + 3)/4 \\ S &= AP \cdot CP / 2 = 3\sqrt{3}/8 \\ R(3\sqrt{3} + 3)/4 &= 3\sqrt{3}/8 \\ R &= 12\sqrt{3}/(8(3\sqrt{3} + 3)) = 3\sqrt{3}/(2(3\sqrt{3} + 3)) = \\ &= 3\sqrt{3}/(6(\sqrt{3} + 1)) = \sqrt{3}/(2(\sqrt{3} + 1)) = \\ &= \sqrt{3}(\sqrt{3} - 1)/(2(\sqrt{3} + 1)(\sqrt{3} - 1)) = \sqrt{3}(\sqrt{3} - 1)/(2(3 - 1)) = \\ &= \sqrt{3}(\sqrt{3} - 1)/4 \end{aligned}$$

$$\begin{aligned} S &= r(CB + CP + BP)/2 = r(1 + \sqrt{3}/2 + 1/2)/2 = \\ &= r(\sqrt{3} + 3)/4 \\ S &= \sqrt{3}/2 \cdot 1/2 = \sqrt{3}/8 \\ r(\sqrt{3} + 3)/4 &= \sqrt{3}/8 \\ r &= 4\sqrt{3}/8(\sqrt{3} + 3) \\ r &= \sqrt{3}/2(\sqrt{3} + 3) = (\sqrt{3} - 3)\sqrt{3}/2(\sqrt{3} + 3)(\sqrt{3} - 3) = \\ &= (3 - \sqrt{3})\sqrt{3}/2(9 - 3) = (3 - \sqrt{3})\sqrt{3}/12 = (\sqrt{3} - 1)/12 = \\ &= (\sqrt{3} - 1)/4 \end{aligned}$$

$$\begin{aligned} O_1O_2 &= \sqrt{2(\sqrt{3}(\sqrt{3} - 1)/4)^2 + 2((\sqrt{3} - 1)/4)^2} = \\ &= \sqrt{2((3 - \sqrt{3})/4)^2 + 2((\sqrt{3} - 1)/4)^2} = \\ &= (1/4)\sqrt{2(3 - \sqrt{3})^2 + 2(\sqrt{3} - 1)^2} = \\ &= (1/4)\sqrt{2(9 - 6\sqrt{3} + 3) + 2(3 - 2\sqrt{3} + 1)} = \\ &= (1/4)\sqrt{2(9 - 6\sqrt{3} + 3 + 3 - 2\sqrt{3} + 1)} = \\ &= \sqrt{2}/4 \cdot \sqrt{2(9 - 6\sqrt{3} + 3 + 3 - 2\sqrt{3} + 1)} = \\ &= 1/4 \cdot \sqrt{2(16 - 8\sqrt{3})} = \\ &= 1/4 \cdot \sqrt{16(2 - \sqrt{3})} = \sqrt{2 - \sqrt{3}} \end{aligned}$$