



Куб суммы и разности (разложить на множители методом группировки)

$$\begin{aligned} 1)(!!!) x^3 + 3x^2y + 3y^2x + y^3 &= x^3 + x^2y + x^2y + x^2y + y^2x + y^2x + y^2x + y^3 \\ &+ y^3 = (x^3 + x^2y) + (y^2x + y^3) + x^2y + x^2y + y^2x + y^2x = x^2(x+y) + \\ &y^2(x+y) + x^2y + x^2y + y^2x + y^2x = (x^2 + y^2)(x+y) + x^2y + x^2y + y^2x + \\ &y^2x = (x^2 + y^2)(x+y) + (x^2y + x^2y + y^2x + y^2x) = (x^2 + y^2)(x+y) + \\ &xy(x+x+y+y) = (x^2 + y^2)(x+y) + 2xy(x+y) = (x^2 + 2xy + y^2)(x+y) = \\ &(x+y)^2(x+y) = (x+y)^3 \end{aligned}$$

$$(x+y)^3 = (x+y)(x+y)(x+y) = x^3 + 3x^2y + 3y^2x + y^3$$

$$\begin{aligned} 2)(!!!) x^3 - 3x^2y + 3xy^2 - y^3 &= x^3 - x^2y - x^2y - x^2y + xy^2 + xy^2 + xy^2 - \\ &y^3 = (x^3 - x^2y) + (xy^2 - y^3) - x^2y - x^2y + xy^2 + xy^2 = x^2(x-y) + y^2(x-y) - \\ &xy(x+x-y-y) = (y^2 + x^2)(x-y) - 2xy(x-y) = (x-y)^3 \end{aligned}$$