

## Куб суммы и разности

$$1)(!!!) x^3 + 3x^2y + 3y^2x + y^3 = x^3 + x^2y + x^2y + x^2y + y^2x + y^2x + y^2x + y^3 =$$

$$x(x^2 + y^2) + y(x^2 + y^2) + xy(x + y) + xy(x + y) = (x + y)(x^2 + y^2) + (xy + xy)(x + y)$$

$$= (x + y)((x^2 + y^2) + (xy + xy)) = (x + y)(x^2 + y^2 + xy + xy) = (x + y)(x + y)(x + y) = (x + y)^3$$

$$2)(!!!) x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - x^2y - x^2y - x^2y + y^2x + y^2x + y^2x - y^3 =$$

$$= x(x^2 + y^2) + y(-x^2 - y^2) + xy(-x + y) + xy(-x + y) = x(x^2 + y^2) - y(x^2 + y^2) + xy(-x + y) + xy(-x + y)$$

$$= (x - y)(x^2 + y^2) + (xy + xy)(-x + y) = (x - y)(x^2 + y^2) - (xy + xy)(x - y) = (x - y)((x^2 + y^2) - (xy + xy))$$

$$= (x - y)(x^2 + y^2 - xy - xy) = (x - y)^3$$

$$x^3 + 3x^2y + 3y^2x + y^3 = (x + y)^3$$

$$x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$$

$$x^2 - xy - xy + y^2 = x(x - y) + y(-x + y) = x(x - y) - y(x - y) = (x - y)(x - y)$$

