

я с друзьями на алгебре



$$\begin{aligned}x^3 + y^3 &= (x+y)(x^2 + y^2 - xy) \\x^3 - y^3 &= (x^2 + xy + y^2)(x-y) \\x^3 + 3x^2y + 3y^2x + y^3 &\Rightarrow (x+y)^3 \\x^3 - 3x^2y + 3y^2x - y^3 &= (x-y)^3\end{aligned}$$

разложить на множители

$$\begin{aligned}5) (*) (x+y+z)^3 - x^3 - y^3 - z^3 &= ((x+y+z)^2 + (x+y+z)x + x^2) \\((x+y+z)x - (y^3+z^3)) &= (2xy+2xz+2zy+x^2+y^2+z^2+ \\x^2+yx+zx+x^2)(y+z) - (z+y)(z^2+y^2-zy) &= \\= (3xy+3xz+2yz+y^2+z^2+3x^2)(y+z) - & \\-(z+y)(z^2+y^2-zy) &= (z+y)((3xy+3xz+2yz+y^2+z^2+3x^2) - \\(z^2+y^2-zy)) &= (z+y)(3xy+3xz+2yz+y^2+z^2+3x^2-z^2-y^2+zy) \\= (z+y)(3xy+3xz+3yz+3x^2) &= 3(z+y)(xy+xz+yz+x^2) \\= 3(z+y)(x(x+y)+z(x+y)) &= 3(z+y)(x+y)(x+z) \\(x+y+z)^2 = (x+y+z)(x+y+z) &= x^2+xy+xz+yx+y^2+yz+zx+yz+z^2 = \\2xy+2xz+2zy+x^2+y^2+z^2 &= \end{aligned}$$

$$\begin{aligned}9) (*) 8x^3 + y^3 + 6y^2 + 12y + 8 &= 8x^3 + y^3 + 3y^2 \cdot 2 + 3 \cdot 2^2 y + 2^3 \\= (2+y)^3 + (2x)^3 &= ((2+y)+2x)((2+y)^2 + (2x)^2 - (2+y)2x) = \\= (2+y+2x)(4+y^2+4y+4x^2-4x-2xy) &= \end{aligned}$$

расщипление одного слагаемого на несколько

1) **доделать...**

$$x^3 + 3x^2y + 3y^2x + y^3 \Rightarrow (x+y)^3$$

$$\begin{aligned}x^3 + y^3 + z^3 - 3xyz &= \underline{x^3 + 3x^2y + 3y^2x + y^3} + z^3 - 3xyz - 3x^2y - 3y^2x = \\= (x+y)^3 + z^3 - 3xyz - 3x^2y - 3y^2x &= (x+y)^3 +\end{aligned}$$

$$\begin{aligned}z^3 - 3xy(z+x+y) &= ((x+y)+z)((x+y)^2 + z^2 - (x+y)z) - 3xy(z+x+y) = (x+y+z)(x^2 + 2xy + y^2 + z^2 - \\2xz - yz) - 3xy(x+y+z) &= (x+y+z)(x^2 + 2xy + y^2 + z^2 - xz - yz - 3xy) = (x+y+z)(x^2 - xy + y^2 + z^2 - \\xz - yz) &= (x+y+z)(x^2 + y^2 + z^2 - xy - xz - yz)\end{aligned}$$

2) **разложить на множители**

$$\begin{aligned}x^5 - y^5 &= x^4y - y^4x + x^4y - x^4y + y^4x - y^4x = x^4(x+y) - y^4(x+y) - yx(x^3 - y^3) = \\= (x+y)(x^4 - y^4) - yx(x^3 - y^3) &= (x+y)(x^2 + y^2)(x+y)(x-y) - yx(x^2 + xy + y^2)(x-y) = \\= (x-y)((x^2 + y^2)(x+y)^2 - yx(x^2 + xy + y^2)) &= \\(x-y)((x^2 + y^2)(x^2 + 2xy + y^2) - (yx)x^2 - (xy)^2 - (xy)y^2) &= (x-y)((x^2 + y^2)x^2 + (x^2 + y^2)2xy + (x^2 + y^2)y^2 - yx^3 - x^2y^2 - xy^3) = \\(x-y)(x^4 + y^2x^2 + 2yx^3 + 2xy^3 + x^2y^2 + y^4 - yx^3 - x^2y^2 - xy^3) &= (x-y)(y^2x^2(1-1) + yx^3(2-1) + xy^3(2-1) + x^4 + x^2y^2 + y^4) = \\(x-y)(yx^3 + xy^3 + x^4 + x^2y^2 + y^4) &= \end{aligned}$$

3) **раскрыть скобки и привести подобные**

$$(x+y)^5 = (x+y)^2 \cdot$$

$$\begin{aligned}(x+y)^3 &= (x^2 + 2xy + y^2)(x^3 + 3x^2y + 3y^2x + y^3) = x^5 + 3x^4y + 3y^2x^3 + x^2y^3 + 2x^4y + 6x^3y^2 + 6y^3x^2 + 2y^4x + \\x^3y^2 + 3y^3x^2 + 3y^4x + y^5 &= x^5 + y^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5y^4x\end{aligned}$$