Возвратные уравнения 4-ой степени

$ax^4 + bx^3 + cx^2 + dx + e = 0$

если $e/a = (d/b)^2$, то делим уравнение на x^2

и делаем замену

1) $x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$

2) $18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$

\+ подумать почему такая замена возможна только при условии $e/a = (d/b)^2$

$ax^4 + bx^3 + cx^2 + dx + e = 0 \ |:x^2$

$ax^2+bx+c+d/x+e/x^2=0;$

$ax^2+e/x^2+bx+d/x+c=0;$

$a(x^2+(e/a)/x^2)+b(x+(d/b)/x)+c=0;$

$y=(x+(d/b)/x);$

$y^2=(x+(d/b)/x)^2;$

$y^2=x^2+2(d/b)+(d^2/b^2)/x^2$

$y^2-2(d/b)=x^2+(d^2/b^2)/x^2$

$(e/a)=(d^2/b^2)$

$x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$

$e/a=1/1=1$

$(d/b)^2=(-7/-7)^2=1^2=1$

$x^4 - 7x^3 + 14x^2 - 7x + 1 = 0 \ | :x^2$

$x^2-7x+14-7/x+1/x^2=0$

$x^2+1/x^2+14-7x-7/x=0$

$x^2+1/x^2+14-7(x+1/x)=0$

$y=(x+1/x)$

$y^2=x^2+2+1/x^2;$

$x^2+1/x^2=y^2-2$

$y^2-2+14-7y=0;$

$y^2-7y+12=0;$

$y1=3;$

$y2=4;$

$x+1/x=3$

$x+1/x-3=0$

$x/1+1/x-3/1=0;$

$(x^2+1-3x)/x=0;$

$x^2+1-3x=0;$

и

$x<>0;$

$x^2-3x+1=0$

$D=9-4=5;D>0;$

$x1=(3-V5)/2$

$x2=(3+V5)/2$

x+1/x=4

x+1/x-4=0

(x^2+1-4x)/x=0;

(x^2+1-4x)=0

и

x<>0;

x^2-4x+1=0

D*=4-1=3; D*>0;

x3=2-V3;

x4=2+V3;

Answer:

2-V3;2+V3;(3-V5)/2;(3+V5)/2

$18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$

$e/a=8/18=4/9$

$(d/b)^2=(2/-3)^2=4/9$

$18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0 \ |:x^2$

$18x^2-3x-25+2/x+8/x^2=0;$

$18x^2+8/x^2-3x+2/x-25=0;$

$2(9x^2+4/x^2)-3x+2/x-25=0;$

$y=(-3x+2/x);$

$y^2=(-3x+2/x)^2;$

$y^2=9x^2-2*3x*2/x+4/x^2;$

$y^2=9x^2-12+4/x^2;$

$y^2+12=9x^2+4/x^2;$

$2(y^2+12)+y-25=0;$

$2y^2+24+y-25=0;$

$2y^2+y-1=0;$

$D=1+8=9;\ D>0;\ VD=3;$

$y1=(-1-3)/4=-1;$

$y2=(-1+3)/4=½;$

$-3x+2/x=-1;$

$-3x+2/x+1=0;$

$-3x^2+2+x=0;$

и

$x<>0;$

$-3x^2+2+x=0;$

$3x^2-x-2=0;$

$x1=1$

$x2=-⅔$

$-3x+2/x=1/2;$

$-3x+2/x-1/2=0;$

$-6x^2+4-x=0;$

и

$x<>0;$

$-6x^2-x+4=0;$

$6x^2+x-4=0;$

$D=1+4*4*6=97;D>0;$

$x3=(-1-V97)/12;$

$x4=(-1+V97)/12;$

$Answer:1;-⅔;(-1-V97)/12;(-1+V97)/12$