

Возвратные уравнения 4-ой степени

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

если $e/a = (d/b)^2$, то делим уравнение на x^2 и делаем замену

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$1/1 = (-7/-7)^2$$

$$1) x^2 - 7x + 14 - 7/x + 1/x^2 = 0$$

$$(-7x - 7/x) + (x^2 + 1/x^2) + 14 = 0$$

$$-7(x + 1/x) + (x^2 + 1/x^2) + 14 = 0$$

$$x + 1/x = t$$

$$x^2 + 2 + 1/x^2 = t^2$$

$$x^2 + 1/x^2 = t^2 - 2$$

$$-7t + (t^2 - 2) + 14 = 0$$

$$-7t + t^2 - 2 + 14 = 0$$

$$t^2 - 7t + 12 = 0$$

$$t_1 = 4$$

$$t_2 = 3$$

$$x + 1/x = 4$$

$$x^2 + 1 - 4x = 0$$

$$D_2 = 16$$

$$x_1 = 2 - \sqrt{3}$$

$$x_2 = 2 + \sqrt{3}$$

$$x + 1/x = 3$$

$$x^2 - 3x + 1 = 0$$

$$D = 5$$

$$x_1 = 3 - \sqrt{5}/2$$

$$x_2 = 3 + \sqrt{5}/2$$

Ответ: $2 - \sqrt{3}$; $3 + \sqrt{5}/2$.

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$

$$8/18 = (2/3)^2$$

$$4/9 = 4/9$$

$$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0$$

$$-3x + 2/x = t$$

$$9x^2 - 12 + 4/x^2 = t^2$$

$$9x^2 + 4/x^2 = t^2 + 12$$

$$2t^2 + 24 + t - 25 = 0$$

$$2t^2 + t - 1 = 0$$

$$D = 1 + 8 = 9$$

$$t_1 = -1 - 3/4 = -1$$

$$t_2 = -1 + 3/4 = 1/4$$

$$-3x + 2/x = -1$$

$$-3x^2 + x + 2 = 0$$

$$x_1 = 1$$

$$x_2 = -2/3$$



$$-3x + 2/x = 1/2$$

$$-3x^2 - 1/2x + 2 = 0$$

$$-6x^2 - x + 4 = 0$$

$$D = 1 + 96 = 97$$

$$x_1 = 1 - \sqrt{97}/12$$

$$x_2 = 1 + \sqrt{97}/12$$