Возвратные уравнения 4-ой степени

$ax^4 + bx^3 + cx^2 + dx + e = 0$

если $e/a = (d/b)^2$, то делим уравнение на x^2

и делаем замену

1) $x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$

2) $18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$

1) $x^4 - 7x^3 + 14x^2 - 7x + 1 = 0 / x^2$ $\quad 1/1=(-7/-7)^2$

$x^2-7x+14-7/x+1/x^2=0$

$x^2+1/x^2 \quad -7x-7/x \quad +14=0$

$\underline{x^2+1/x^2} \quad -7\underline{(x+1/x)} \quad +14=0$

$(x+1/x)=y$

$(x+1/x)^2=y^2$

$x^2+1/x^2=y^2-2$

$y^2-7y+12=0$

$D=49-48=1$

$y_1=3$

$y_2=4$

$x+1/x=3 / {*x}$

$x^2+1=3x$

$x^2-3x+1=0$

$D=9-4=5$

$x_1=(3+\sqrt{5})/2$

$x_2=(3-\sqrt{5})/2$

$x^2-4x+1=0$

$x_3=(4+2\sqrt{3})/2$

$x_4=(4-2\sqrt{3})/2$

2) $18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0 / x^2$

$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0$

$8/18=(2/-3)^2$

$4/9=4/9$

$18x^2 + 8/x^2 -3x+2/x -25=0$

$2(9x^2 +4/x^2)-(3x-2/x)-25=0$

$3x-2/x=y$

$(3x-2/x)^2=y^2$

$(3x-2/x)^2=9x^2-12+4/x^2=y^2$

$9x^2+4/x^2=y^2+12$

$2(y^2+12)-y-25=0$

$2y^2-y-1=0$

$D=1+8=9$

$y_1=(1+3)/4=1$

$y_2=(1-3)/4=-1/2$

$3x-2/x=1 / {*x}$

$3x^2-2=x$

$3x^2-x-2=0$

$x_1=1$

$x_2=-2/3$

$3x-2/x=-1/2$

$6x^2-4=-x$

$6x^2+x-4=0$

$D=1+96=97$

$x_3=(-1-\sqrt{97})/12$

$x_4=(-1+\sqrt{97})/12$

ДОКАЗАТЕЛЬСТВО

$ax^4 + bx^3 + cx^2 + dx + e = 0/x^2$

$ax^2 + bx+c+d/x+e/x^2=0$

$ax^2 +e/x^2 \quad + bx+d/x \quad +c=0$

$a(x^2 +(a/e)(1/x^2)) \quad + b(x+(d/b)(1/x)) \quad +c=0$

$x+(d/b)(1/x)=y$

$(x+(d/b)(1/x))^2=y^2$

$(x+(d/b)(1/x))^2=x^2+2(d/b)+(d/b)^2(1/x^2)=y^2$

$\underline{x^2+(d/b)^2(1/x^2)=y^2-2(d/b)}$

$\underline{x^2+(e/a)(1/x^2)=y^2-2(d/b)}$

$y^2-2(d/b) + b*y + c=0$

$ax^2+bx+c=0$

$x_1+x_2=-b/a$

$x_1*x_2=c/a$

$x_1=1$

$1*x_2=c/a$