

Возвратные уравнения 4-ой степени

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

если $e/a = (d/b)^2$, то делим уравнение на x^2 и делаем

замену

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$x^2 - 7x + 14 - 7/x + 1/x^2 = 0$$

$$x^2 + 1/x^2 - 7x - 7/x + 14 = 0$$

$$x^2 + 1/x^2 - 7(x + 1/x) + 14 = 0$$

$$x + 1/x = y$$

$$(x + 1/x)^2 = y^2$$

$$x^2 + 2 + 1/x^2 = y^2$$

$$x^2 + 1/x^2 = y^2 - 2$$

$$y^2 - 7y + 12 = 0$$

$$y^2 = 3$$

$$y_1 = 4$$

$$x + 1/x = 3$$

$$x^2/x + 1/x = 3x/x$$

$$(x^2 + 1) = 3x$$

$$x^2 - 3x + 1 = 0$$

$$d = 9 - 4 = 5$$

$$x_1 = (3 - \sqrt{5})/2$$

$$x_2 = (3 + \sqrt{5})/2$$

$$x^2 + 1 = 4x$$

$$x^2 - 4x + 1$$

$$d = 16 - 4 = 12$$

$$x_1 = 2 - \sqrt{3}$$

$$x_2 = 2 + \sqrt{3}$$

$$\text{ответ: } (3 + \sqrt{5})/2, (3 - \sqrt{5})/2, 2 - \sqrt{3}, 2 + \sqrt{3}$$

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0 \text{ дз}$$

$$8/18 = (2/3)^2$$

$$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0$$

$$18x^2 + 8/x^2 - 3x + 2/x - 25 = 0$$

$$y = 2/x - 3x$$

$$y^2 = (2/x - 3x)^2$$

$$y^2 = 4/x^2 - 12 + 9x^2$$

$$y^2 + 12 = 4/x^2 + 9x^2$$

$$2(y^2 + 12) + y - 25 = 0$$

$$2y^2 + 24 + y - 25 = 0$$

$$2y^2 + y - 1 = 0$$

$$d = 1 + 8 = 9$$

$$ax^2 + bx + c = 0 \quad X_1 + X_2 = -B/A \quad X_1 * X_2 = C/A$$

теорема о сумме коэф-тов

если сумма коэф-тов = 0, то один из корней 1, а второй c/a

$$2 - 1 - 1 = 0$$

$$y_1 = (-1 + 3)/4 = 1/2$$

$$y_2 = -1$$



$$313123121231X^2 - 313123121230X - 1 = 0$$

$$X_1 = 1$$

$$X_2 = -1 / 313123121231$$

$$2/x - 3x = -1$$

$$2/x - 3x + 1 = 0$$

$$(2 - 3x^2 + x)/x = 0$$

$$x \neq 0$$

$$2 - 3x^2 + x = 0$$

$$3x^2 - x - 2 = 0$$

$$x_1 = 1$$

$$x_2 = -2/3$$

$$2/x - 3x = 1/2$$

$$2/x - 3x - 1/2 = (4 - 6x^2 - 1x)/2x = 0$$

$$6x^2 + x - 4 = 0$$

$$d = 1 + 96 = 97$$

$$x_1 = (-1 + \sqrt{97})/12$$

$$x_2 = (-1 - \sqrt{97})/12$$

$$\text{ответ: } 1, -2/3, (-1 + \sqrt{97})/12, (-1 - \sqrt{97})/12.$$

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

$$ax^2 + bx + c + d/x + e/x^2 = 0$$

$$ax^2 + e/x^2 + bx + d/x + c = 0$$

$$bx + d/x = y$$

$$(bx + d/x)^2 = y^2$$

$$b^2x^2 + 2bd + d^2/x^2 = y^2$$

$$b^2x^2 + d^2/x^2 = y^2 - 2bd$$

$$b^2(x^2 + (d^2/b^2) * 1/x^2) = y^2 - 2bd$$

$$a(x^2 + e/a * 1/x^2) + y + c = 0$$

$$(d^2/b^2) = e/a$$