

Возвратные уравнения 4-ой степени

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

если $e/a = (d/b)^2$, то делим уравнение на x^2 и делаем замену

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$1/1 = (-7/-7)^2$$

$$x^2 - 7x + 14 - 7/x + 1/x^2 = 0$$

$$x^2 + 1/x^2 - 7(x + 1/x) + 14 = 0$$

$$x + 1/x = y$$

$$x^2 + 2 + 1/x^2 = y^2$$

$$x^2 + 1/x^2 = y^2 - 2$$

$$y^2 - 2 - 7y + 14 = 0$$

$$y^2 - 7y + 12 = 0$$

$$y_1 = 4$$

$$y_2 = 3$$

$$x + 1/x - 4 = 0$$

$$x^2 - 4x + 1 = 0$$

$$x + 1/x - 3 = 0$$

$$x^2 - 3x + 1 = 0$$

$$D = 4 - 1 = 3$$

$$D = 9 - 4 = 5$$

$$x_1 = (2 - \sqrt{3})/1$$

$$x_2 = (2 + \sqrt{3})/1$$

$$x_3 = (3 - \sqrt{5})/2$$

$$x_4 = (3 + \sqrt{5})/2$$

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$

$$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0$$

$$18x^2 + 8/x^2 - 3x + 2/x - 25 = 0$$

$$-3x + 2/x = t$$

$$(-3x + 2/x)^2 = t^2$$

$$9x^2 - 12 + 4/x^2 = t^2$$

$$9x^2 + 4/x^2 = t^2 + 12$$

$$2(t^2 + 12) + t - 25 = 0$$

$$2t^2 + 24 + t - 25 = 0$$

$$2t^2 + t - 1 = 0$$

$$D = 1 + 8 = 9$$

$$t_1 = (-1 - 3)/4 = -1$$

$$t_2 = (-1 + 3)/4 = 1/2$$

$$-3x + 2/x + 1 = 0$$

$$3x - 2/x - 1 = 0$$

$$3x^2 - x - 2 = 0$$

$$D = 1 + 24 = 25$$

$$x_1 = (1 - 5)/6 = -2/3$$

$$x_2 = (1 + 5)/6 = 1$$

$$-3x + 2/x - 1/2 = 0$$

$$3x - 2/x + 1/2 = 0$$

$$6x^2 + x - 4 = 0$$

$$D = 1 + 96 = 97$$

$$x_1 = (-1 + \sqrt{97})/12$$

$$x_2 = (-1 - \sqrt{97})/12$$



$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

$$ax^2 + bx + c + d/x + e/x^2 = 0$$

$$ax^2 + e/x^2 + bx + d/x + c = 0$$

$$a(x^2 + e/a * 1/x^2) + b(x + d/b * 1/x) + c = 0$$

$$y = x + d/b * 1/x$$

$$y^2 = x^2 + 2d/b + d^2/b^2 * 1/x^2$$

$$y^2 - 2d/b = x^2 + d^2/b^2 * 1/x^2$$

$$e/a = (d/b)^2$$

Ответ: $-2/3; 1; (-1 + \sqrt{97})/12; (-1 - \sqrt{97})/12$