

Возвратные уравнения 4-ой степени

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

если $e/a = (d/b)^2$, то делим уравнение на x^2

и делаем замену

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$



+ подумать почему такая замена возможна только при условии $e/a = (d/b)^2$

$$ax^4 + bx^3 + cx^2 + dx + e = 0 \quad | :x^2$$

$$ax^2 + bx + c + d/x + e/x^2 = 0;$$

$$ax^2 + e/x^2 + bx + d/x + c = 0;$$

$$a(x^2 + (e/a)/x^2) + b(x + (d/b)/x) + c = 0;$$

$$y = (x + (d/b)/x);$$

$$y^2 = (x + (d/b)/x)^2;$$

$$y^2 = x^2 + 2(d/b) + (d^2/b^2)/x^2$$

$$y^2 - 2(d/b) = x^2 + (d^2/b^2)/x^2$$

$$(e/a) = (d^2/b^2)$$

$$x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$e/a = 1/1 = 1$$

$$(d/b)^2 = (-7/-7)^2 = 1^2 = 1$$

$$x^4 - 7x^3 + 14x^2 - 7x + 1 = 0 \quad | :x^2$$

$$x^2 - 7x + 14 - 7/x + 1/x^2 = 0$$

$$x^2 + 1/x^2 + 14 - 7x - 7/x = 0$$

$$x^2 + 1/x^2 + 14 - 7(x + 1/x) = 0$$

$$y = (x + 1/x)$$

$$y^2 = x^2 + 2 + 1/x^2;$$

$$x^2 + 1/x^2 = y^2 - 2$$

$$y^2 - 2 + 14 - 7y = 0;$$

$$y^2 - 7y + 12 = 0;$$

$$y_1 = 3;$$

$$y_2 = 4;$$

$$x + 1/x = 3$$

$$x + 1/x - 3 = 0$$

$$x/1 + 1/x - 3/1 = 0;$$

$$(x^2 + 1 - 3x)/x = 0;$$

$$x^2 + 1 - 3x = 0;$$

и $x <> 0;$

$$x^2 - 3x + 1 = 0$$

$$D = 9 - 4 = 5; D > 0;$$

$$x_1 = (3 - \sqrt{5})/2$$

$$x_2 = (3 + \sqrt{5})/2$$

$$x + 1/x = 4$$

$$x + 1/x - 4 = 0$$

$$(x^2 + 1 - 4x)/x = 0;$$

$$(x^2 + 1 - 4x) = 0$$

и $x <> 0;$

$$x^2 - 4x + 1 = 0$$

$$D^* = 4 - 1 = 3; D^* > 0;$$

$$x_3 = 2 - \sqrt{3};$$

$$x_4 = 2 + \sqrt{3};$$

Answer:
 $2 - \sqrt{3}; 2 + \sqrt{3}; (3 - \sqrt{5})/2; (3 + \sqrt{5})/2$

$$18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$

$$e/a = 8/18 = 4/9$$

$$(d/b)^2 = (2/-3)^2 = 4/9$$

$$18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0 \quad | :x^2$$

$$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0;$$

$$18x^2 + 8/x^2 - 3x + 2/x - 25 = 0;$$

$$2(9x^2 + 4/x^2) - 3x + 2/x - 25 = 0;$$

$$y = (-3x + 2/x);$$

$$y^2 = (-3x + 2/x)^2;$$

$$y^2 = 9x^2 - 2 \cdot 3x \cdot 2/x + 4/x^2;$$

$$y^2 = 9x^2 - 12 + 4/x^2;$$

$$y^2 + 12 = 9x^2 + 4/x^2;$$

$$2(y^2 + 12) + y - 25 = 0;$$

$$2y^2 + 24 + y - 25 = 0;$$

$$2y^2 + y - 1 = 0;$$

$$D = 1 + 8 = 9; D > 0; VD = 3;$$

$$y_1 = (-1 - 3)/4 = -1;$$

$$y_2 = (-1 + 3)/4 = 1/2;$$

$$-3x + 2/x = -1;$$

$$-3x + 2/x + 1 = 0;$$

$$-3x^2 + 2 + x = 0;$$

и $x <> 0;$

$$-3x^2 + 2 + x = 0;$$

$$3x^2 - x - 2 = 0;$$

$$x_1 = 1$$

$$x_2 = -2/3$$

$$-3x + 2/x = 1/2;$$

$$-3x + 2/x - 1/2 = 0;$$

$$-6x^2 + 4 - x = 0;$$

и $x <> 0;$

$$-6x^2 - x + 4 = 0;$$

$$6x^2 + x - 4 = 0;$$

$$D = 1 + 4 \cdot 4 \cdot 6 = 97; D > 0;$$

$$x_3 = (-1 - \sqrt{97})/12;$$

$$x_4 = (-1 + \sqrt{97})/12;$$

Answer: $1; -2/3; (-1 - \sqrt{97})/12; (-1 + \sqrt{97})/12$