

Возвратные уравнения 4-ой степени

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

если $e/a = (d/b)^2$, то делим уравнение на x^2 и делаем замену

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$

$$1) x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

$$e/a = (d/b)^2$$

$$1/1 = (-7/-7)^2 = 1$$

$$x^2 - 7x + 14 - 7/x + 1/x^2 = 0$$

$$-7x - 7/x + x^2 + 1/x^2 + 14 = 0$$

$$-7(x+1/x) + x^2 + 1/x^2 + 14 = 0$$

$$x+1/x = t$$

$$(x+1/x)^2 = t^2$$

$$x^2 + 2x \cdot 1/x + 1/x^2 = t^2$$

$$x^2 + 2 + 1/x^2 = t^2$$

$$x^2 + 1/x^2 = t^2 - 2$$

$$-7(x+1/x) + x^2 + 1/x^2 + 14 = 0$$

$$-7(t) + t^2 - 2 + 14 = 0$$

$$t^2 - 7t + 12 = 0$$

$$t_1 t_2 = 12$$

$$t_1 + t_2 = 7$$

3 4



$$x+1/x = 3$$

$$x+1/x - 3 = 0$$

$$x^2 - 3x + 1 = 0$$

$$D = 9 - 4 = 5$$

$$x_1 = (3 + \sqrt{5})/2$$

$$x_2 = (3 - \sqrt{5})/2$$

$$x+1/x = 4$$

$$x + 1/x - 4 = 0$$

$$x^2 - 4x + 1 = 0$$

$$D^* = 2^2 - 1 = 3$$

$$x_1 = 2 + \sqrt{3}$$

$$x_2 = 2 - \sqrt{3}$$

$$\text{answer: } (3 + \sqrt{5})/2 ; (3 - \sqrt{5})/2 ; 2 + \sqrt{3} ; 2 - \sqrt{3}$$

$$2) 18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0$$

$$e/a = (d/b)^2$$

$$8/18 = (2/-3)^2$$

$$4/9 = 4/9$$

$$18x^4 - 3x^3 - 25x^2 + 2x + 8 = 0 / x^2$$

$$18x^2 - 3x - 25 + 2/x + 8/x^2 = 0$$

$$18x^2 + 8x^2 - 3x + 2/x - 25 = 0$$

$$t = -3x + 2/x$$

$$t^2 = (-3x + 2/x)^2$$

$$t^2 = 9x^2 + 2(-3x)(2/x) + 4/x^2$$

$$t^2 = 9x^2 - 12 + 4/x^2$$

$$t^2 + 12 = 9x^2 + 4/x^2$$

$$2(t^2 + 12) + t - 25 = 0$$

$$2t^2 + 24 + t - 25 = 0$$

$$2t^2 + t - 1 = 0$$

$$D = 1 - 4(2)(-1) = 1 + 8 = 9$$

$$t_1 = (-1 + 3)/4 = 1/2$$

$$t_2 = (-1 - 3)/4 = -1$$

$$-3x + 2/x = 1/2$$

$$-6x^2 + 4 = x$$

$$-6x^2 - x + 4 = 0$$

$$6x^2 + x - 4 = 0$$

$$D = 1 - -96 = 97$$

$$x_1 = (-1 + \sqrt{97})/12$$

$$x_2 = (-1 - \sqrt{97})/12$$

$$ax^2 + bx + c = 0$$

$$x_1 \cdot x_2 = c/a$$

$$x_1 + x_2 = -b/a$$

$$-3x + 2/x = -1$$

$$-3x^2 + 2 + x = 0$$

$$3x^2 - x - 2 = 0$$

$$\text{т.к. } 3 + (-1) + (-2) = 0, \text{ то}$$

$$x_1 = 1$$

$$x_1 \cdot x_2 = c/a$$

$$1 \cdot x_2 = c/a$$

$$x_2 = c/a = -2/3$$

$$\text{answer: } -2/3 ; (-1 - \sqrt{97})/12 ; (-1 + \sqrt{97})/12 ; 1$$