

### Однородные уравнения

Однородные уравнения - это уравнения, все члены которых имеют одинаковую степень, а справа 0.

Уравнение вида  $Au^2 + Buv + Cv^2 = 0$  называется однородным уравнением II-ой степени относительно U и V.

Проверяем возможность деления на U и V.

Делим на  $U^2(V^2)$

$AU^2 + BUV + CV^2 = 0$  делим на  $U^2(U \neq 0)$ , получаем

$$A + BV/U + CV^2/U^2 = 0$$

Пусть  $V/U = y$ , тогда  $V^2/U^2 = y^2$ , получаем ур-ие:

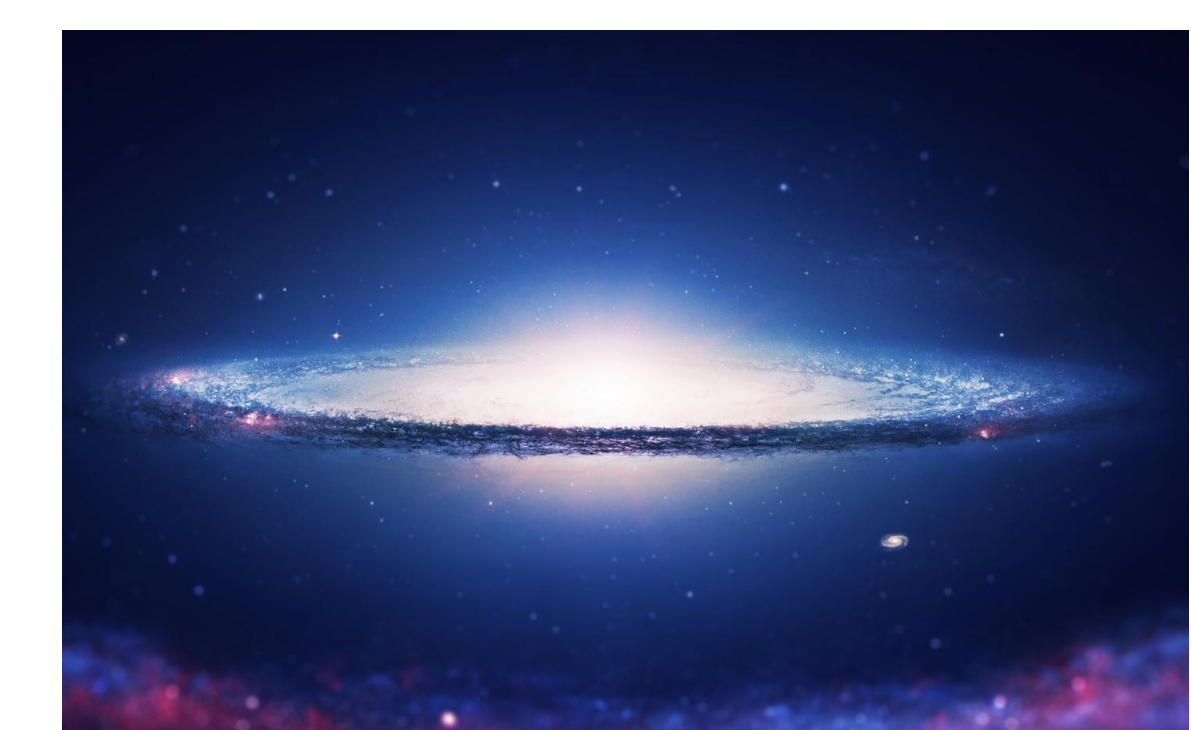
$$A + By + Cy^2 = 0$$

### Обратная замена

Задачи на однородные уравнения

$$1) (x^2 - x + 1)^4 - 10x^2(x^2 - x + 1)^2 + 9x^4 = 0$$

$$2) 2(x - 1)^4 - 5(x^2 - 3x + 2)^2 + 2(x - 2)^4 = 0$$



$A^*U^3 + BU^2V + CUV^2 + DV^3 = 0$  | : $U^3$   
 $A + BV/U + CV^2/U^2 + DV^3/U^3 = 0$  |  
 $V/U = y$   
 $A + By + Cy^2 + Dy^3 = 0$  |

$$(x^2 - x + 1)^4 - 10x^2(x^2 - x + 1)^2 + 9x^4 = 0$$

$$u = (x^2 - x + 1)^2$$

$$v = x^2$$

$$u^2 - 10vu + 9v^2 = 0;$$

$$\text{пусть } u=0, \text{ тогда } 0^2 - 10v \cdot 0 + 9v^2 = 0; \Leftrightarrow 9v^2 = 0 \Leftrightarrow v = 0 \Rightarrow$$

$$x^2 = 0 \Leftrightarrow x = 0$$

$$(x^2 - x + 1)^2 = 0 \Leftrightarrow x^2 - x + 1 = 0 \Rightarrow 0^2 - 0 + 1 = 1 \neq 0 \text{ неверно}$$

$$(U \neq 0)$$

$$u^2 - 10vu + 9v^2 = 0; | :u^2$$

$$1 - 10v/u + 9v^2/u^2 = 0;$$

$$y = v/u;$$

$$y^2 = v^2/u^2;$$

$$1 - 10y + 9y^2 = 0;$$

$$9y^2 - 10y + 1 = 0;$$

$$y_1 = 1;$$

$$y_2 = 1/9;$$

$$v/u = 1$$

$$x^2/(x^2 - x + 1)^2 = 1;$$

$$x^2/(x^2 - x + 1)^2 - 1 = 0;$$

$$(x^2 - (x^2 - x + 1)^2)/(x^2 - x + 1)^2 = 0;$$

$$(x^2 - (x^2 - x + 1)^2) = 0;$$

и

$$(x^2 - x + 1)^2 < 0;$$

$$(x^2 - (x^2 - x + 1)^2) = 0;$$

$$(x - (x^2 - x + 1))(x + (x^2 - x + 1)) = 0;$$

$$(x - (x^2 - x + 1)) = 0;$$

$$x - x^2 + x - 1 = 0;$$

$$x^2 - 2x + 1 = 0;$$

$$x_1 = 1;$$

$$x_2 = 1;$$

$$(x + (x^2 - x + 1)) = 0;$$

$$x + x^2 - x + 1 = 0;$$

$$x^2 + 1 = 0;$$

$$x^2 = -1$$

no solutions

$$x^2 - 3x + 2 = 0$$

$$x_1, x_2$$

$$x^2 - 3x + 2 = a(x - x_1)(x - x_2)$$

$$x^2/(x^2 - x + 1)^2 = 1/9;$$

$$x^2/(x^2 - x + 1)^2 - 1/9 = 0;$$

$$(9x^2 - (x^2 - x + 1)^2)/9(x^2 - x + 1)^2 = 0;$$

$$(9x^2 - (x^2 - x + 1)^2) = 0;$$

and

$$9(x^2 - x + 1)^2 < 0;$$

$$(9x^2 - (x^2 - x + 1)^2) = 0;$$

$$(3x - (x^2 - x + 1))(3x + (x^2 - x + 1)) = 0;$$

$$(3x - (x^2 - x + 1)) = 0;$$

$$3x - x^2 + x - 1 = 0;$$

$$x^2 - 4x + 1 = 0;$$

$$D^* = (-4)^2 - 4 \cdot 1 = 3; D^* > 0;$$

$$x_1 = 2 - \sqrt{3};$$

$$x_2 = 2 + \sqrt{3};$$

$$(3x + (x^2 - x + 1)) = 0;$$

$$3x + x^2 - x + 1 = 0;$$

$$x^2 + 2x + 1 = 0;$$

$$x_3 = -1;$$

$$x_4 = -1;$$

$$x^2 - x + 1 = 0;$$

$$D = 1 - 4 = -3; D < 0;$$

no solutions

$$\text{Answer: } 1; 1; 2 - \sqrt{3}; 2 + \sqrt{3}; -1; -1;$$

$$(x - 1)(x - 2) = 0$$

and

$$(x - 1)(x - 3) = 0$$

$$x = 1 \rightarrow \text{answer}$$

$$2(x - 1)^4 - 5(x^2 - 3x + 2)^2 + 2(x - 2)^4 = 0$$

$$(x^2 - 3x + 2) = 0$$

$$x_1 = 1$$

$$x_2 = 2$$

$$(x^2 - 3x + 2) = 1(x - 1)(x - 2)$$

$$2(x - 1)^4 - 5(x - 1)^2(x - 2)^2 + 2(x - 2)^4 = 0;$$

$$u = (x - 1)^2;$$

$$v = (x - 2)^2;$$

$$2u^2 - 5uv + 2v^2 = 0;$$

$$\text{пусть } u=0, \text{ тогда } 2 \cdot 0 - 5 \cdot 0 \cdot v + 2v^2 = 0; \Leftrightarrow 2v^2 = 0; \Leftrightarrow v = 0;$$

$$\text{so that } (x - 2) = 0; \text{ therefore } x = 2;$$

$$\text{and } (x - 1) = 0; \text{ therefore } x = 1;$$

$$(u \neq 0)$$

$$2u^2 - 5uv + 2v^2 = 0; | :u^2$$

$$2 - 5v/u + 2v^2/u^2 = 0;$$

$$y = v/u;$$

$$2 - 5y + 2y^2 = 0;$$

$$2y^2 - 5y + 2 = 0;$$

$$D = 25 - 16 = 9; D > 0; VD = 3;$$

$$y_1 = (5 - 3)/4 = 1/2;$$

$$y_2 = (5 + 3)/4 = 2;$$

$$v/u = 1/2;$$

$$(x - 2)^2/(x - 1)^2 = 1/2;$$

$$(x - 2)^2/(x - 1)^2 - 1/2 = 0;$$

$$((x - 2)/(x - 1))^2 - V(1/2)^2 = 0$$

$$((x - 2)/(x - 1) - V(1/2))((x - 2)/(x - 1) + V(1/2)) = 0;$$

$$((x - 2)/(x - 1) - V(1/2)) = 0;$$

or

$$((x - 2)/(x - 1) + V(1/2)) = 0;$$

$$((x - 2)/(x - 1) - V(1/2)) = 0;$$

$$(x - 2)/(x - 1) - 1/\sqrt{2} = 0;$$

$$(x - 2)/(x - 1) - \sqrt{2}/2 = 0;$$

$$(2(x - 2) - \sqrt{2}(x - 1))/(2(x - 1)) = 0;$$

$$2(x - 2) - \sqrt{2}(x - 1) = 0;$$

and

$$2(x - 1) < 0; \Leftrightarrow x < 1$$

$$2(x - 2) - \sqrt{2}(x - 1) = 0;$$

$$2x - 4 - \sqrt{2}x + \sqrt{2} = 0;$$

$$2x - \sqrt{2}x = 4 - \sqrt{2};$$

$$x(2 - \sqrt{2}) = 4 - \sqrt{2}; | : (2 - \sqrt{2})$$

$$x = (4 - \sqrt{2}) / (2 - \sqrt{2});$$

$$x = (4 - \sqrt{2})(2 + \sqrt{2}) / ((2 - \sqrt{2})(2 + \sqrt{2}));$$

$$x = (8 + 4\sqrt{2} - 2\sqrt{2} - \sqrt{2}^2) / (2^2 - \sqrt{2}^2);$$

$$x = (8 + 2\sqrt{2} - 2) / (4 - 2);$$

$$x = (6 + 2\sqrt{2}) / 2;$$

$$x = 3 + \sqrt{2}; \Rightarrow x < 1$$

$$((x - 2)/(x - 1) + V(1/2)) = 0;$$

$$(x - 2)/(x - 1) + \sqrt{2}/2 = 0;$$

$$(2(x - 2) + \sqrt{2}(x - 1))/(2(x - 1)) = 0;$$

$$2(x - 2) + \sqrt{2}(x - 1) = 0;$$

and

$$2(x - 1) < 0; \Leftrightarrow x < 1$$

$$2(x - 2) + \sqrt{2}(x - 1) = 0;$$

$$2x - 4 + \sqrt{2}x - \sqrt{2} = 0;$$

$$2x + \sqrt{2}x = 4 + \sqrt{2};$$

$$x(2 + \sqrt{2}) = 4 + \sqrt{2}; | : (2 + \sqrt{2})$$

$$x = (4 + \sqrt{2}) / (2 + \sqrt{2});$$

$$x = (8 + 4\sqrt{2} + 2\sqrt{2} - \sqrt{2}^2) / (2^2 - \sqrt{2}^2);$$

$$x = (6 - 2\sqrt{2}) / (4 - 2);$$

$$x = 3 - \sqrt{2}; \Rightarrow x < 1$$

$$v/u = 2;$$

$$(x - 2)^2/(x - 1)^2 = 2;$$

$$(x - 2)^2/(x - 1)^2 - 2 = 0;$$

$$(x - 2)^2/(x - 1)^2 - (2)^2 = 0;$$

$$((x - 2)/(x - 1) - 2)((x - 2)/(x - 1) + 2) = 0;$$

$$(x - 2)/(x - 1) - 2 = 0;$$

or

$$(x - 2)/(x - 1) + 2 = 0;$$

$$(x - 2)/(x - 1) - 2 = 0;$$

$$((x - 2) - 2(x - 1))/(x - 1) = 0;$$

$$((x - 2) - 2(x - 1)) = 0;$$

and

$$(x - 1) < 0; \Leftrightarrow x < 1$$

$$(x - 2) - 2(x - 1) = 0;$$

$$x - 2 - 2x + 2 = 0;$$

$$x - 2x = 2 - 2;$$

$$x(1 - 2) = 2 - 2; | : (1 - 2);$$

$$x = (2 - 2) / (1 - 2);$$

$$x = (2 - 2)(1 + 2) / ((1 + 2)(1 - 2));$$

$$X = (2 + 2\sqrt{2} - \sqrt{2} - 2) / (1 - 2);$$

$$x = \sqrt{2} / -1;$$

$$x = -\sqrt{2}; \text{ therefore } x < 1$$

$$(x - 2)/(x - 1) + 2 = 0;$$

$$((x - 2) + 2(x - 1))/(x - 1) = 0;$$

$$((x - 2) + 2(x - 1)) = 0;$$

and

$$(x - 1) < 0; \Leftrightarrow x < 1$$

$$(x - 2) + 2(x - 1) = 0;$$

$$x - 2 + 2x - 2 = 0;$$

$$x + 2x = 2 + 2;$$

$$x(1 + 2) = 2 + 2; | : (1 + 2);$$

$$x = (2 + 2) / (1 + 2);$$

$$x = (2 + 2)(1 - 2) / ((1 + 2)(1 - 2));$$

$$X = (2 - 2\sqrt{2} + \sqrt{2} - 2) / (1 - 2);$$

$$x = -\sqrt{2} / -1;$$

$$x = \sqrt{2}; \text{ therefore } x < 1$$

Answer:

$$3 + \sqrt{2};$$

$$3 - \sqrt{2};$$

$$-\sqrt{2};$$

$$\sqrt{2}$$