

### Однородные уравнения

Однородные уравнения - это уравнения, все члены которых имеют одинаковую степень, а справа 0.

Уравнение вида  $Au^2 + Buv + Cv^2 = 0$  называется однородным уравнением II-ой степени относительно U и V.

Проверяем возможность деления на U и V.

Делим на  $U^2(V^2)$

$AU^2 + BUV + CV^2 = 0$  делим на  $U^2(U! = 0)$ , получаем

$$A + BV/U + CV^2/U^2 = 0$$

Пусть  $V/U = y$ , тогда  $V^2/U^2 = y^2$ , получаем ур-ие:

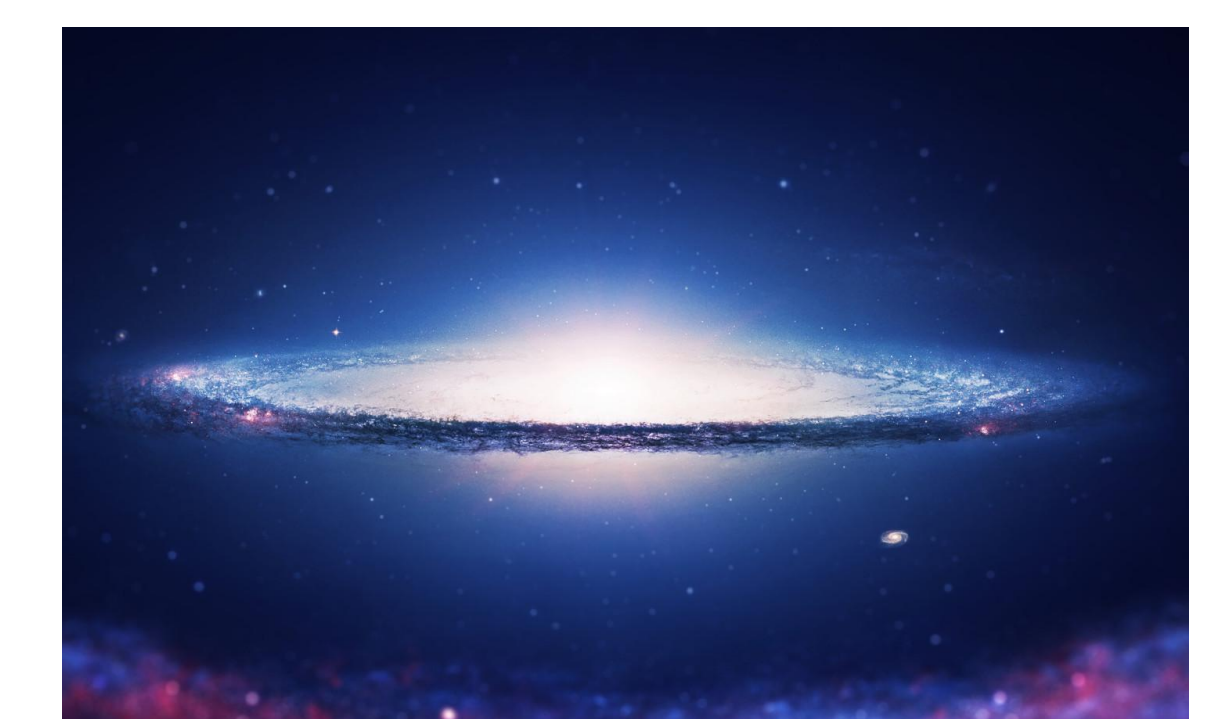
$$A + By + Cy^2 = 0$$

Обратная замена

Задачи на однородные уравнения

1)  $(x^2 - x + 1)^4 - 10x^2(x^2 - x + 1)^2 + 9x^4 = 0$

2)  $2(x - 1)^4 - 5(x^2 - 3x + 2)^2 + 2(x - 2)^4 = 0$



$A*U^3 + BU^2V + CUV^2 + DV^3 = 0$  |  $U^3$   
 $A + BV/U + CV^2/U^2 + DV^3/U^3 = 0$  |  
 $V/U = y$   
 $A + By + Cy^2 + Dy^3 = 0$  |

$$(x^2 - x + 1)^4 - 10x^2(x^2 - x + 1)^2 + 9x^4 = 0$$

$$u = (x^2 - x + 1)^2$$

$$v = x^2$$

$$u^2 - 10vu + 9v^2 = 0;$$

пусть  $u=0$ , тогда  $0^2 - 10v*0 + 9v^2 = 0$ ;  $\Leftrightarrow 9v^2 = 0 \Leftrightarrow v = 0 \Rightarrow$

$$x^2 = 0 \Leftrightarrow x = 0$$

$$(x^2 - x + 1)^2 = 0 \Leftrightarrow x^2 - x + 1 = 0 \Rightarrow 0^2 - 0 + 1 = 0 \text{ неверно}$$

$(U! = 0)$

$$u^2 - 10vu + 9v^2 = 0; |u^2$$

$$1 - 10v/u + 9v^2/u^2 = 0;$$

$$y = v/u;$$

$$y^2 = v^2/u^2;$$

$$1 - 10y + 9y^2 = 0;$$

$$9y^2 - 10y + 1 = 0;$$

$$y1 = 1;$$

$$y2 = 1/9;$$

$$v/u = 1$$

$$x^2/(x^2 - x + 1)^2 = 1;$$

$$x^2/(x^2 - x + 1)^2 - 1 = 0;$$

$$(x^2 - (x^2 - x + 1)^2)/(x^2 - x + 1)^2 = 0;$$

$$(x^2 - (x^2 - x + 1)^2) = 0;$$

и

$$(x^2 - x + 1)^2 <> 0;$$

$$(x^2 - (x^2 - x + 1)^2) = 0;$$

$$(x - (x^2 - x + 1))(x + (x^2 - x + 1)) = 0;$$

$$(x - (x^2 - x + 1)) = 0;$$

$$x - x^2 + x - 1 = 0;$$

$$x^2 - 2x + 1 = 0;$$

$$x1 = 1;$$

$$x2 = 1;$$

$$(x + (x^2 - x + 1)) = 0;$$

$$x + x^2 - x + 1 = 0;$$

$$x^2 + 1 = 0;$$

$$x^2 = -1$$

no solutions

$$x^2 - 3x + 2 = 0$$

$$x1, x2$$

$$x^2 - 3x + 2 = a(x - x1)(x - x2)$$

$$x^2/(x^2 - x + 1)^2 = 1/9;$$

$$x^2/(x^2 - x + 1)^2 - 1/9 = 0;$$

$$(9x^2 - (x^2 - x + 1)^2)/9(x^2 - x + 1)^2 = 0;$$

$$(9x^2 - (x^2 - x + 1)^2) = 0;$$

and

$$9(x^2 - x + 1)^2 <> 0;$$

$$(9x^2 - (x^2 - x + 1)^2) = 0;$$

$$(3x - (x^2 - x + 1))(3x + (x^2 - x + 1)) = 0;$$

$$(3x - (x^2 - x + 1)) = 0;$$

$$3x - x^2 + x - 1 = 0;$$

$$x^2 - 4x + 1 = 0;$$

$$D^* = (-4/2)^2 - 1 = 4 - 1 = 3; D^* > 0;$$

$$x1 = 2 - \sqrt{3};$$

$$x2 = 2 + \sqrt{3};$$

$$(3x + (x^2 - x + 1)) = 0;$$

$$3x + x^2 - x + 1 = 0;$$

$$x^2 + 2x + 1 = 0;$$

$$x3 = -1;$$

$$x4 = -1;$$

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$$x^2 - x + 1 = 0;$$

$$D = 1 - 4 = -3; D < 0;$$

no solutions

Answer: 1; 1; 2-√3; 2+√3; -1; -1;

$$(x-1)(x-2) = 0$$

and

$$(x-1)(x-3) = 0$$

x=1 -> answer

$$2(x - 1)^4 - 5(x^2 - 3x + 2)^2 + 2(x - 2)^4 = 0$$

$$(x^2 - 3x + 2) = 0$$

$$x1 = 1$$

$$x2 = 2$$

$$(x^2 - 3x + 2) = 1(x-1)(x-2)$$

$$2(x - 1)^4 - 5(x-1)^2(x-2)^2 + 2(x - 2)^4 = 0;$$

$$u = (x-1)^2;$$

$$v = (x-2)^2;$$

$$2u^2 - 5uv + 2v^2 = 0;$$

пусть  $u=0$ , тогда  $2*0 - 5*0v + 2v^2 = 0$ ;  $\Leftrightarrow 2v^2 = 0$ ;  $\Leftrightarrow v = 0$ ;

so that  $(x-2) = 0$ ; therefore  $x = 2$ ;

and  $(x-1) = 0$ ; therefore  $x = 1$ ;

$(u! = 0)$

$$2u^2 - 5uv + 2v^2 = 0; |u^2$$

$$2 - 5v/u + 2v^2/u^2 = 0;$$

$$y = v/u;$$

$$2 - 5y + 2y^2 = 0;$$

$$2y^2 - 5y + 2 = 0;$$

$$D = 25 - 16 = 9; D > 0; VD = 3;$$

$$y1 = (5-3)/4 = 1/2;$$

$$y2 = (5+3)/4 = 2;$$

$$v/u = 1/2;$$

$$(x-2)^2/(x-1)^2 = 1/2;$$

$$(x-2)^2/(x-1)^2 - 1/2 = 0$$

$$((x-2)/(x-1))^2 - V(1/2) = 0$$

$$((x-2)/(x-1) - V(1/2))((x-2)/(x-1) + V(1/2)) = 0;$$

$$((x-2)/(x-1) - V(1/2)) = 0;$$

or

$$((x-2)/(x-1) + V(1/2)) = 0;$$

$$((x-2)/(x-1) - V(1/2)) = 0;$$

$$(x-2)/(x-1) - 1/\sqrt{2} = 0;$$

$$(x-2)/(x-1) - \sqrt{2}/2 = 0;$$

$$(2(x-2) - \sqrt{2}(x-1))/(2(x-1)) = 0;$$

$$2(x-2) - \sqrt{2}(x-1) = 0;$$

and

$$2(x-1) <> 0; \Leftrightarrow x <> 1$$

$$2(x-2) - \sqrt{2}(x-1) = 0;$$

$$2x - 4 - \sqrt{2}x + \sqrt{2} = 0;$$

$$2x - \sqrt{2}x = 4 - \sqrt{2};$$

$$x(2 - \sqrt{2}) = 4 - \sqrt{2}; |:(2 - \sqrt{2})$$

$$x = (4 - \sqrt{2})/(2 - \sqrt{2});$$

$$x = (4 - \sqrt{2})(2 + \sqrt{2}) / ((2 - \sqrt{2})(2 + \sqrt{2}));$$

$$x = (8 + 4\sqrt{2} - 2\sqrt{2} - \sqrt{2}^2) / (2^2 - \sqrt{2}^2);$$

$$x = (8 + 2\sqrt{2} - 2) / (4 - 2);$$

$$x = (6 + 2\sqrt{2}) / 2;$$

$$x = 3 + \sqrt{2}; \Rightarrow x <> 1$$

$$((x-2)/(x-1) + V(1/2)) = 0;$$

$$(x-2)/(x-1) + \sqrt{2}/2 = 0;$$

$$(2(x-2) + \sqrt{2}(x-1))/(2(x-1)) = 0;$$

$$2(x-2) + \sqrt{2}(x-1) = 0;$$

and

$$2(x-1) <> 0; \Leftrightarrow x <> 1$$

$$2(x-2) + \sqrt{2}(x-1) = 0;$$

$$2x - 4 + \sqrt{2}x - \sqrt{2} = 0;$$

$$2x + \sqrt{2}x = 4 + \sqrt{2};$$

$$x(2 + \sqrt{2}) = 4 + \sqrt{2}; |:(2 + \sqrt{2})$$

$$x = (4 + \sqrt{2})/(2 + \sqrt{2});$$

$$x = (4 + \sqrt{2})(2 - \sqrt{2})/((2 + \sqrt{2})(2 - \sqrt{2}));$$

$$x = (8 - 4\sqrt{2} + 2\sqrt{2} - \sqrt{2}^2)/(2^2 - \sqrt{2}^2);$$

$$x = (6 - 2\sqrt{2})/(4 - 2);$$

$$x = 3 - \sqrt{2}; \Rightarrow x <> 1$$

$$V(1/2) = V1/\sqrt{2} = 1/\sqrt{2} = \sqrt{2}/(\sqrt{2} * \sqrt{2}) = \sqrt{2}/2$$

Answer:

**3 + √2;**  
**3 - √2;**  
**-√2;**  
**√2**

$$v/u = 2;$$

$$(x-2)^2/(x-1)^2 = 2;$$

$$(x-2)^2/(x-1)^2 - 2 = 0;$$

$$(x-2)^2/(x-1)^2 - (V2)^2 = 0;$$

$$((x-2)/(x-1) - (V2))((x-2)/(x-1) + (V2)) = 0;$$

$$(x-2)/(x-1) - V2 = 0;$$

or

$$(x-2)/(x-1) + V2 = 0;$$

$$(x-2)/(x-1) - V2 = 0;$$

$$((x-2) - V2(x-1))/(x-1) = 0;$$

$$((x-2) - V2(x-1)) = 0;$$

and

$$(x-1) <> 0; \Leftrightarrow x <> 1$$

$$(x-2) - V2(x-1) = 0;$$

$$x - 2 - V2x + V2 = 0;$$

$$x - V2x = 2 - V2;$$

$$x(1 - V2) = 2 - V2; |:(1 - V2);$$

$$x = (2 - V2)/(1 - V2);$$

$$x = (2 - V2)(1 + V2)/((1 - V2)(1 + V2));$$

$$x = (2 - 2V2 - V2^2 + 2)/(1 - 2);$$

$$x = V2/-1;$$

$$x = -V2; \text{ therefore } x <> 1$$

$$(x-2)/(x-1) + V2 = 0;$$

$$((x-2) + V2(x-1))/(x-1) = 0;$$

$$((x-2) + V2(x-1)) = 0;$$

and

$$(x-1) <> 0; \Leftrightarrow x <> 1$$

$$(x-2) + V2(x-1) = 0;$$

$$x - 2 + V2x - V2 = 0;$$

$$x + V2x = 2 + V2;$$

$$x(1 + V2) = 2 + V2; |:(1 + V2);$$

$$x = (2 + V2)/(1 + V2);$$

$$x = (2 + V2)(1 - V2)/((1 + V2)(1 - V2));$$

$$x = (2 - 2V2 + V2^2 - 2)/(1 - 2);$$

$$x = -V2/-1;$$

$$x = V2; \text{ therefore } x <> 1$$