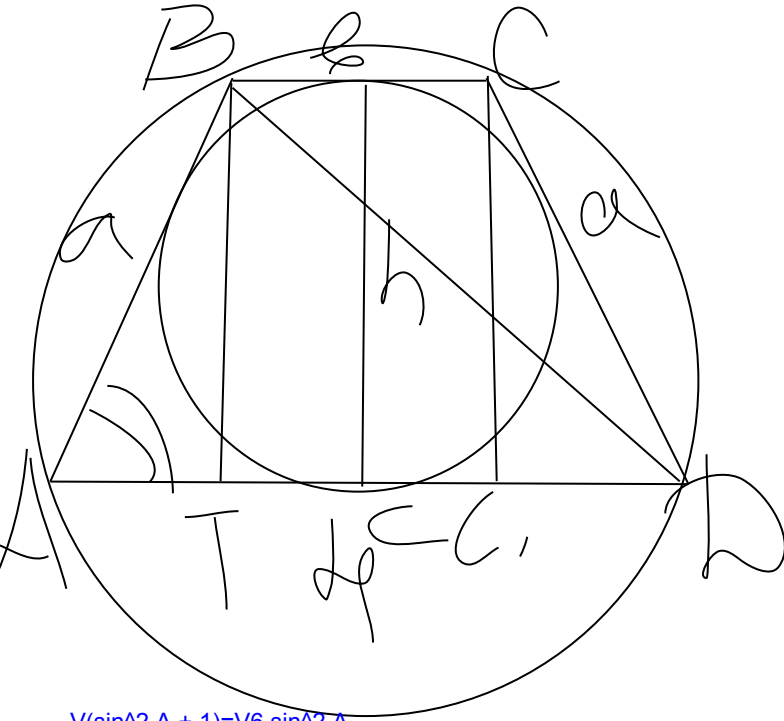


Дана равнобочная трапеция, вписанная и описанная окружностями.
 Отношение высоты к радиусу описанной окружности равно $\sqrt{2/3}$. Найти углы трапеции



$$\begin{aligned} \sqrt{\sin^2 A + 1} &= \sqrt{6} \sin^2 A \\ \sqrt{\sin^2 A + 1} &^2 = 6 \sin^4 A \\ \sin^2 A + 1 &= 6 \sin^4 A \\ \sin^2 A &= t \\ t + 1 &= 6t^2 \\ 6t^2 - t - 1 &= 0 \\ D &= 1 + 24 = 25 \\ t &= \frac{1+5}{12} = \frac{1}{2}; -\frac{1}{3} \\ \sin^2 A &= \frac{1}{2} \\ \sin A &= \frac{\sqrt{2}}{2} \\ A &= \frac{\pi}{6} + 2\pi k \\ A &= \frac{\pi}{5} + 2\pi k \end{aligned}$$

$$\begin{aligned} A &= \frac{7\pi}{6} + 2\pi k \\ A &= \frac{5\pi}{6} + 2\pi k \end{aligned}$$

$$\begin{aligned} A &= \frac{\pi}{6} \\ B &= \frac{\pi}{6} \\ \text{Answer: } A &= D = \frac{\pi}{6}; B = C = \frac{5\pi}{6} \end{aligned}$$

$$\begin{aligned} BT &= h \\ \frac{h}{R} &= \sqrt{\frac{2}{3}} \Rightarrow R = \frac{h\sqrt{3}}{\sqrt{2}} \\ BC + AD &= x = AB + CD \\ AB &= CD = \frac{x}{2} \end{aligned}$$

$$\begin{aligned} \text{в } \triangle BAT \\ \sin A &= \frac{BT}{AB} = \frac{BT}{x/2} = \frac{2BT}{x} = \frac{2h}{x} \\ x \sin A &= 2h \Rightarrow h = \frac{x \sin A}{2} \end{aligned}$$

$$\frac{BD}{\sin A} = 2R \Rightarrow R = \frac{BD}{2 \sin A}$$

$$\begin{aligned} \text{в } \triangle BTD \text{ по Т Пифагора} \\ BD^2 &= h^2 + TD^2 = h^2 + \\ &+ \left(\frac{b+c}{2}\right)^2 = h^2 + \left(\frac{b+c}{2}\right)^2 = h^2 + \frac{x^2}{4} \\ BD &= \sqrt{h^2 + \frac{x^2}{4}} \end{aligned}$$

$$\frac{BD}{\sin A} = 2R$$

$$\frac{\sqrt{h^2 + \frac{x^2}{4}}}{\sin A} = 2R$$

$$\frac{\sqrt{x^2 \sin^2 A + \frac{x^2}{4}}}{\sin A} = 2R$$

$$\frac{x}{2} \cdot \sqrt{\sin^2 A + 1} / \sin A = 2R$$

$$R = \frac{h\sqrt{3}}{\sqrt{2}} = \frac{x \sin A}{2} \cdot \frac{\sqrt{3}}{\sqrt{2}} = \frac{x\sqrt{3} \sin A}{2\sqrt{2}}$$

$$\frac{x}{2} \cdot \sqrt{\sin^2 A + 1} / \sin A = 2 \cdot \frac{x\sqrt{3} \sin A}{2\sqrt{2}}$$

$$\frac{x}{2} \cdot \sqrt{\sin^2 A + 1} / \sin A = \frac{x\sqrt{3} \sin A}{\sqrt{2}}$$

$$\frac{1}{2} \cdot \sqrt{\sin^2 A + 1} / \sin A = \frac{\sqrt{3} \sin A}{\sqrt{2}}$$

$$\sqrt{\sin^2 A + 1} = 2\sqrt{3} \sin^2 A / \sqrt{2}$$

$$\sqrt{\sin^2 A + 1} = \sqrt{6} \sin^2 A$$