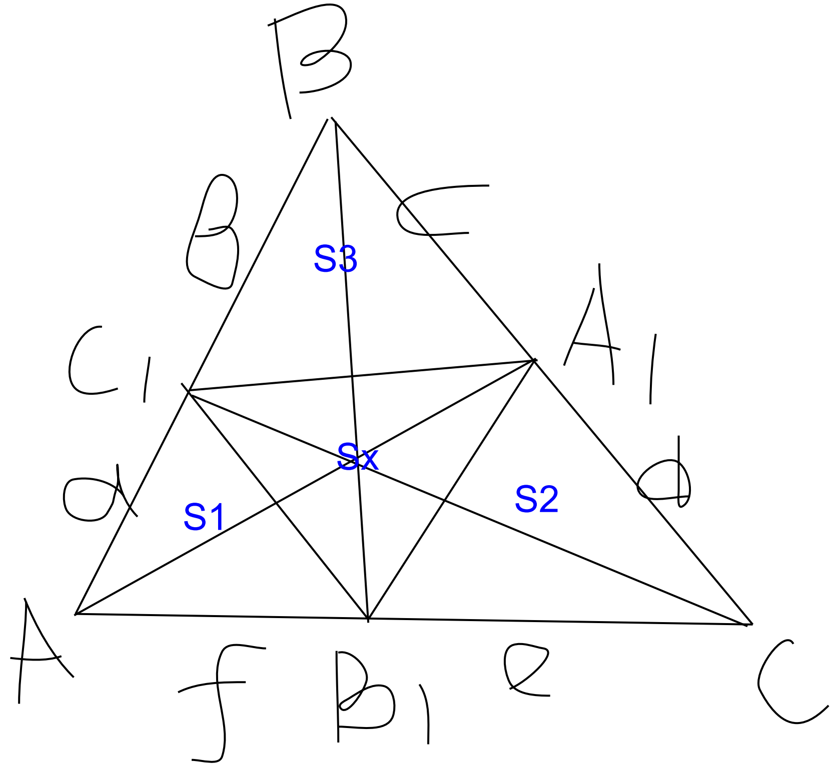
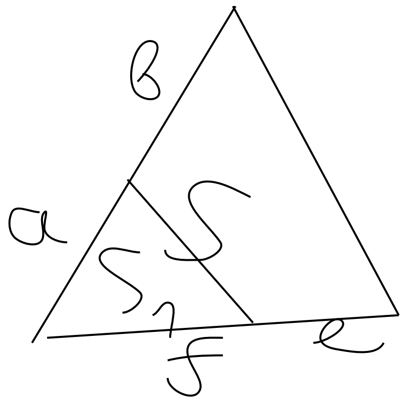
Точка A1,B1,C1 лежат соответственно на сторонах BC,AC и AB трABC, причём AA1,BB1,CC1 пересекаются в одной точке. Доказать, что S(трA1B1C1)<=1/4*S(трABC)





S1=S(AC1B1)
S2=S(CA1B1)
S3=S(BC1A1)
Sx=S(A1B1C1)
S=S(ABC)
Sx<=½ S
AC1/BC1*BA1/CA1*CB1/AB1=1
a/b*c/d*e/f=1=>ace=bdf

AC1/BC1 = S(ACC1)/S(CC1B)

S1/S= 1/2sinA * a*f / 1/2sinA (a+b)(f+e)=
=af/((a+b)(f+e))

S2/S=1/2sinB *c*b / 1/2sinB(a+b)(c+d)=
=cb/(a+b)(c+d)

S3/S=de/(c+d)(f+e)

S1+S2+S3+Sx=S
Sx<=1/4S
Sx=S-(S1+S2+S3)
S-(S1+S2+S3)<=1/4S
S-(Saf/((a+b)(f+e))+Scb/(a+b)(c+d)+Sde/(c+d)(f+e))<=1/4S
1-(af/((a+b)(f+e))+cb/(a+b)(c+d)+de/(c+d)(f+e))<=1/4

1-(af/((a+b)(f+e))+cb/(a+b)(c+d)+de/(c+d)(f+e))<=¼
1-¼<=af/((a+b)(f+e))+cb/(a+b)(c+d)+de/(c+d)(f+e)
¾<=[af(c+d)+cb(f+e)+de(a+b)]/((a+b)(f+e)(c+d))
¾<=[afc+afd+cbf+cbe+dea+deb]/((a+b)(f+e)(c+d))
3(a+b)(f+e)(c+d)<=4[afc+afd+cbf+cbe+dea+deb]
3(afc+afd+aec+aed+bfc+bfd+bec+bed)<=4[afc+afd+cbf+cbe+dea+deb]
3afc+3afd+3aec+3aed+3bfc+3bfd+3bec+3bed<=4afc+4afd+4cbf+4cbe+4dea+4deb
0<=afc+afd+cbf+cbe+dea+deb-3aec-3bdf
0<=afc+afd+cbf+cbe+dea+deb-6aec
6aec<=afc+afd+cbf+cbe+dea+deb
6<=afc/aec+afd/bdf+cbf/bdf+cbe/aec+dea/aec+deb/bdf
6<=f/e+a/b+c/d+b/a+d/c+e/f
6<=(b/a+a/b)+(c/d+d/c)+(e/f+f/e)

(b/a+a/b)=(b^2+a^2)/ab>=2

(b^2+a^2)/ab>=2
a^2+b^2>=2ab
(a-b)^2>=0