

Дан произвольный трABC, в который вписана окружность радиуса r. В углы этого треугольника вписаны окружности, касающиеся сторон этого треугольника (прилежащих к этим углам) и вписанной окружности. Радиусы этих окружностей для углов A,B,C r1,r2,r3 соответственно. Радиусы r1,r2,r3 известны. Найти r.

$$1 + \operatorname{tg}^2 z = 1 + \frac{\sin^2 z}{\cos^2 z}$$

$$z = \frac{(\cos^2 z + \sin^2 z)}{\cos^2 z} = \frac{1}{\cos^2 z}$$

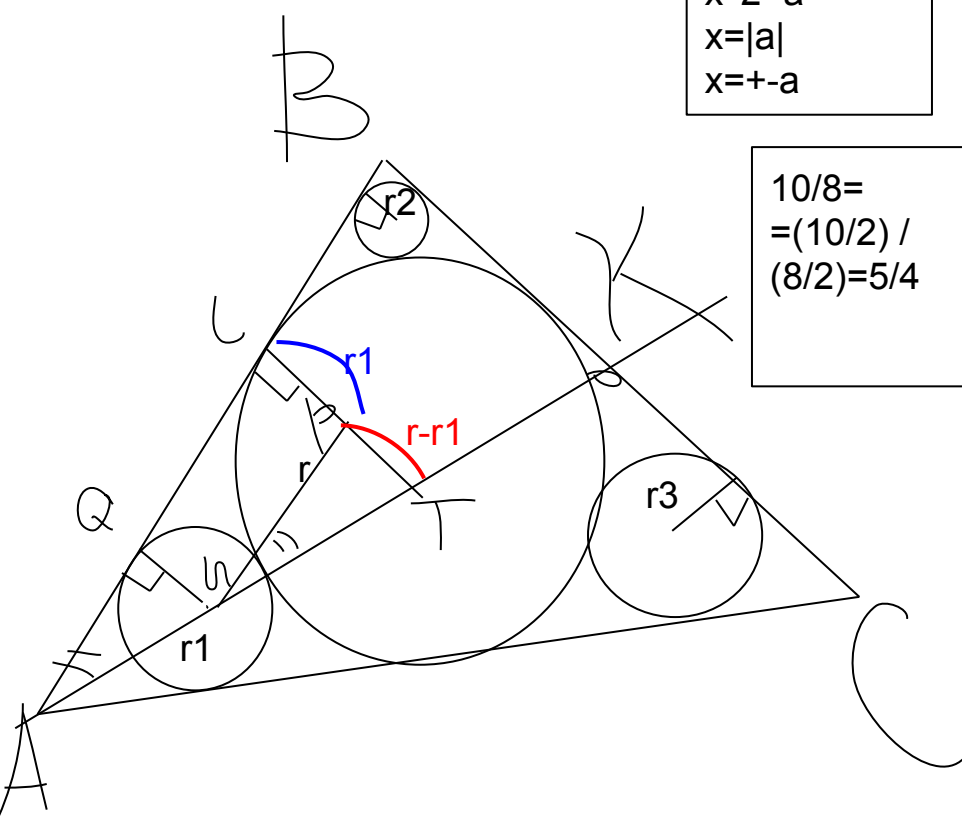
$$x^2 = a$$

$$x = |a|$$

$$x = \pm a$$

$$\frac{10}{8} = \frac{(10/2)}{(8/2)} = \frac{5}{4}$$

every cloud has a silver lining



AK - биссектриса => уг BAK = A/2
 рассмотрим тр-к PUT
 $\sin(A/2) = PT/UT = (r-r_1)/(r+r_1)$
 $\sin(A/2)(r+r_1) = (r-r_1)$
 $\sin(A/2)r + \sin(A/2)r_1 = r - r_1$
 $\sin(A/2)r_1 + r_1 = r - \sin(A/2)r$
 $r_1(\sin(A/2) + 1) = r(1 - \sin(A/2))$
 $r_1 = r(1 - \sin(A/2)) / (\sin(A/2) + 1)$
 $r_2 = r(1 - \sin(B/2)) / (\sin(B/2) + 1)$
 $r_3 = r(1 - \sin(C/2)) / (\sin(C/2) + 1)$
 $A + B + C = 180$
 $A/2 + B/2 + C/2 = 90$
 $A/2 = x, B/2 = y, C/2 = z$

$$r_1 = r(1 - \sin x) / (\sin x + 1) \Rightarrow r_1/r = (1 - \sin x) / (\sin x + 1)$$

$$r_2 = r(1 - \sin y) / (\sin y + 1)$$

$$r_3 = r(1 - \sin z) / (\sin z + 1)$$

$$x + y + z = 90$$

$$\frac{(1 - \sin x) / (\sin x + 1)}{(1 - \sin y) / (\sin y + 1)} = \frac{(1 - 2\sin(x/2)\cos(x/2)) / \cos^2(x/2)}{(1 - 2\sin(y/2)\cos(y/2)) / \cos^2(y/2)} = \frac{(1/\cos^2(x/2) - 2\operatorname{tg}(x/2)) / (2\operatorname{tg}(x/2) + 1/\cos^2(x/2))}{(1/\cos^2(y/2) - 2\operatorname{tg}(y/2)) / (2\operatorname{tg}(y/2) + 1/\cos^2(y/2))} = \frac{(1 + \operatorname{tg}^2(x/2) - 2\operatorname{tg}(x/2)) / (2\operatorname{tg}(x/2) + 1 + \operatorname{tg}^2(x/2))}{(1 + \operatorname{tg}^2(y/2) - 2\operatorname{tg}(y/2)) / (2\operatorname{tg}(y/2) + 1 + \operatorname{tg}^2(y/2))} = \frac{(\operatorname{tg}(x/2) - 1)^2 / (\operatorname{tg}(x/2) + 1)^2}{(\operatorname{tg}(y/2) - 1)^2 / (\operatorname{tg}(y/2) + 1)^2}$$

$$r_1/r = (\operatorname{tg}(x/2) - 1)^2 / (\operatorname{tg}(x/2) + 1)^2$$

$$V(r_1/r) = |(\operatorname{tg}(x/2) - 1) / (\operatorname{tg}(x/2) + 1)| = -(\operatorname{tg}(x/2) - 1) / (\operatorname{tg}(x/2) + 1) = (1 - \operatorname{tg}(x/2)) / (\operatorname{tg}(x/2) + 1) = (\operatorname{tg}(45) - \operatorname{tg}(x/2)) / (1 + \operatorname{tg}(x/2)\operatorname{tg}(45)) = \operatorname{tg}(P/4 - x/2)$$

$$0 < A < 180$$

$$0 < A/2 = x < 90$$

$$0 < x/2 < 45$$

$$0 < \operatorname{tg} x/2 < 1$$

$$V(r_1/r) = \operatorname{tg}(P/4 - x/2)$$

$$V(r_2/r) = \operatorname{tg}(P/4 - y/2)$$

$$V(r_3/r) = \operatorname{tg}(P/4 - z/2)$$

$$P/4 - x/2 + P/4 - y/2 + P/4 - z/2 = 3P/4 - (x+y+z)/2 = P/2$$

$$\operatorname{ctg}(P/2) = 0 \quad \cos(P/2) = 0$$

$$\operatorname{ctg}(u+w+q) = \frac{\cos(u+w+q)}{\sin(u+w+q)} = \frac{\cos((u+w)+q)}{\sin((u+w)+q)} = \frac{(\cos(u+w)\cos q - \sin(u+w)\sin q)}{(\sin(u+w)\cos q + \cos(u+w)\sin q)} = \frac{((\cos u \cos w - \sin u \sin w)\cos q - (\sin u \cos w + \sin w \cos u)\sin q)}{((\sin u \cos w + \sin w \cos u)\cos q + (\cos u \cos w - \sin u \sin w)\sin q)} = \frac{(\cos u \cos w \cos q - \sin u \sin w \cos q - \sin u \cos w \sin q - \sin w \cos u \sin q)}{(\sin u \cos w \cos q + \sin w \cos u \cos q + \cos u \cos w \sin q - \sin u \sin w \sin q)} = \frac{(\cos u \cos w \cos q - \sin u \sin w \cos q - \sin u \cos w \sin q - \sin w \cos u \sin q)}{(\cos u \cos w \cos q)} \cdot \frac{(\sin u \cos w \cos q + \sin w \cos u \cos q + \cos u \cos w \sin q - \sin u \sin w \sin q)}{(\sin u \cos w \cos q + \sin w \cos u \cos q + \cos u \cos w \sin q - \sin u \sin w \sin q)} = \frac{(1 - \sin u \sin w) / (\cos u \cos w) - (\sin u \sin q) / (\cos u \cos q) - (\sin w \sin q) / (\cos w \cos q)}{((\sin u / \cos u + \sin w / \cos w + \sin q / \cos q) - (\sin u \sin w \sin q) / (\cos u \cos w \cos q))} = \frac{(1 - \operatorname{tgu} \operatorname{tgw} - \operatorname{tgu} \operatorname{tgq} - \operatorname{tgw} \operatorname{tgq})}{(\operatorname{tgu} + \operatorname{tgw} + \operatorname{tgq} - \operatorname{tgu} \operatorname{tgw} \operatorname{tgq})}$$

$$0 = (1 - V(r_1/r) \cdot V(r_2/r) - V(r_1/r) \cdot V(r_3/r) - V(r_2/r) \cdot V(r_3/r)) / (V(r_1/r) + V(r_2/r) + V(r_3/r) - V(r_1/r) \cdot V(r_2/r) \cdot V(r_3/r))$$

$$1 - V(r_1/r) \cdot V(r_2/r) - V(r_1/r) \cdot V(r_3/r) - V(r_2/r) \cdot V(r_3/r) = 0$$

$$r - V(r_1 \cdot r_2) - V(r_1 \cdot r_3) - V(r_2 \cdot r_3) = 0$$

$$r = V(r_1 \cdot r_2) + V(r_1 \cdot r_3) + V(r_2 \cdot r_3)$$

$$\operatorname{tg}(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{(\sin a \cos b + \sin b \cos a)}{(\cos a \cos b - \sin a \sin b)} = \frac{(\sin a \cos b + \sin b \cos a) / (\cos a \cos b)}{((\cos a \cos b - \sin a \sin b) / (\cos a \cos b))} = \frac{(\sin a / \cos a + \sin b / \cos b)}{(1 - (\sin a \sin b / \cos a \cos b))} = \frac{(\operatorname{tga} + \operatorname{tgb})}{(1 - \operatorname{tga} \operatorname{tgb})}$$

$$\operatorname{tg}(a-b) = \frac{(\operatorname{tga} - \operatorname{tgb})}{(1 + \operatorname{tga} \operatorname{tgb})}$$