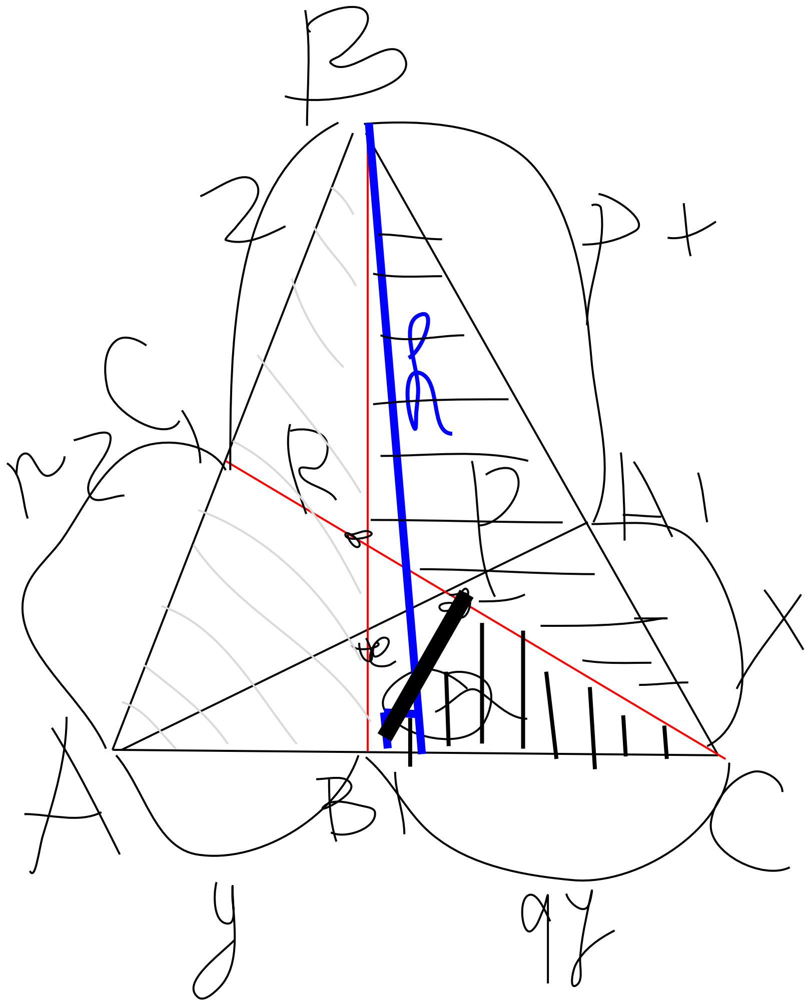
Дан трABC BA1/A1C=p, CB1/B1A=q, AC1/C1B=r. Найти S(трPQR)/S(трABC)



1)S(PQR)/S(ABC)=?

S(ABC)=w=S(ABB1)+S(BB1C)

AB1/B1C=1/q => S(ABB1)/S(BB1C)=
=AB1*h/2 / B1C*h/2 = 1/q

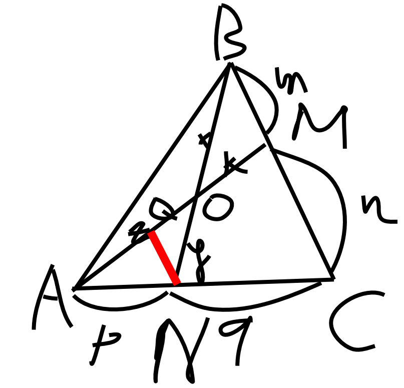
S(BB1C)=? (через w)
S(ABB1)=a
S(BB1C)=b
S(ABB1)+S(BB1C)=w
S(ABB1)/S(BB1C)=1/q
a+b=w
a/b=1/q

a=w-b
(w-b)/b=1/q

(w-b)/b-1/q=0
((w-b)q-b)/bq=0
wq-bq-b=0
wq=b(q+1)
b=wq/(q+1)
S(BB1C)=wq/(q+1)

во всем тр-ке ABC по T о ТОПОТЕ:
BR/RB1=z/rz (1+y/qy)=1/r*(q+1)/q
BR/RB1=k => S(CBR)/S(CRB1)=k

идея откусывания



ТОПОТ
x/y=m/n * (1+q/p)
z/k=p/q * (1+n/m)

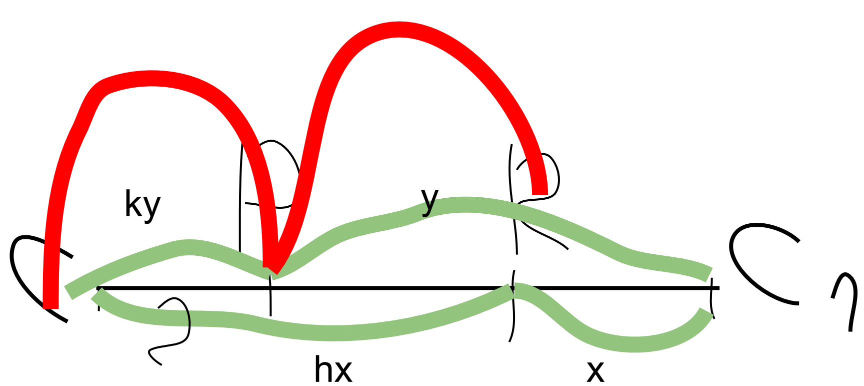
2)S(CBR)/S(CRB1)=1/r*(q+1)/q
S(CBR)+S(CRB1)=wq/(q+1)
S(CBR)=s
S(CRB1)=f
s/f=1/r*(q+1)/q
s+f=wq/(q+1)

s=fq/r(q+1)
fq/r(q+1)+f=wq/(q+1)

f(r(q+1)+q)/r(q+1)=wq/(q+1)
f=wqr(q+1)/(q+1)(r(q+1)+q)
f=wqr/(r(q+1)+q)
S(CRB1)=wqr/(r(q+1)+q)
CR/RC1=CB1/B1A*(1+AC1/C1B)=qy/y*(1+rz/z)=q*(1+r)=h
CP/PC1=CA1/A1B*(1+BC1/C1A)=x/px*(1+z/rz)=1/p(1+1/r)=k

3)CR/RC1=h
CP/PC1=k
CP/PR=ky/ (y-x)=
=ky/(y-(ky+y)/(h+1))=
=k/[1 - (k+1)/(h+1)]=
=k/[(h+1 - k - 1) / (h+1)] =
=k/ (h+k) /(h+1)=(hk+h) / (h+k)=
=h(k+1)/(h+k)

ky+y=hx+x=> x=(ky+y)/(h+1)



4)CP/PR=(hk+h) / (h+k)
CP/PR=(q*(1+r) * 1/p(1+1/r) + q*(1+r)) / (q*(1+r) + 1/p(1+1/r))=
=((q+rq) * r/p(r+1) + q+rq) / (q+rq + r/p(1+r))=
=(qr + r^2q + qpr + qp + r^2qp + rqp)/p(r+1) / (qp + rqp + rqp + r^2qp + r)/p(1+r)=
=(qr + r^2q + 2qpr + qp + r^2qp) / (qp + 2rqp + r^2qp + r)

CP/PR=q*(1+r)[1/p(1+1/r) + 1]/[q*(1+r) + 1/p(1+1/r)]=
=q*(1+r)[r/(p(1+r)) + 1]/[q*(1+r) + r/(p(1+r))]=
=q*(1+r)[r + p(1+r)]/p(1+r) / [q*(1+r)p(1+r) + r]/p(1+r)=
=q*(1+r)[r + p(1+r)] / [qp*(1+r)^2 + r] = S(CPB1)/S(RPB1)
S(CPB1)+S(RPB1) =S(CRB1) = wqr/(r(q+1)+q)