

В трABC имеем $AB=BC$, угол $B = b$. Биссектриса AD угла A пересекает описанную около трABC окружность в точке E . Найти $S(\text{тр}BDE)/S(\text{тр}ABC)$

tip01 выразить угол BAD через b

tip03 достроим CE

tip05 найти отношение $S(ABC)/S(ADC)$

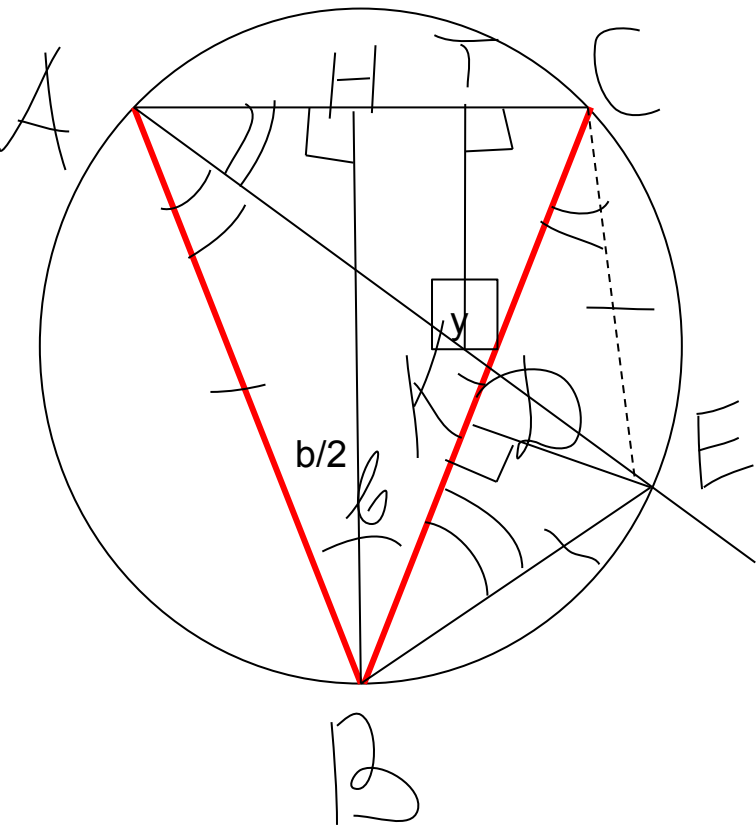
tip02 выразить угол AEB через b

tip04 достроим высоту BH в тр ABC

tip06 $ADC \sim BED$ (2 уг)

tip02_2 $AEB = ACB$

tip07 в тр ABD распиши теорему синусов



$S(\text{тр}BDE)/S(\text{тр}ABC) = ?$

$$1) \angle BAD = \frac{1}{2} \cdot (180 - b) / 2 = (180 - b) / 4$$

$$2) \angle AEB = \angle A = \angle C = (180 - b) / 2$$

$$\begin{aligned} \angle DBE &= 180 - \angle BDE - \angle BEA = 180 - ((180 + 3b) / 4 + (180 - b) / 2) = \\ &= 180 - (((180 + 3b) + 2(180 - b)) / 4) = \\ &= (740 - 540 - b) / 4 = (180 - b) / 4 \end{aligned}$$

$$\begin{aligned} \angle BDE &= \angle ADC = 180 - \angle DAC - \angle C = 180 - (180 - b) / 4 - (180 - b) / 2 = \\ &= 180 - 3(180 - b) / 4 = (720 - 540 + 3b) / 4 = (180 + 3b) / 4 \end{aligned}$$

$$3) \angle BCE = \angle CBE \Rightarrow CE = BE$$

$$S(ABC) = AC \cdot BH / 2 = 2 \operatorname{tg}(b/2) \cdot HB \cdot BH / 2 = \operatorname{tg}(b/2) \cdot BH^2$$

$$\operatorname{tg}(b/2) = AH / HB = AC / (2HB)$$

$$AC = 2 \operatorname{tg}(b/2) \cdot HB$$

$$AD \text{ - биссектр } \Rightarrow BD / AB = DC / AC$$

$$DB / DC = AB / AC = AB / (2AH) = \frac{1}{2} (AB / AH) = 1 / (2 \sin(b/2))$$

$$DB / DC = 1 / (2 \sin(b/2))$$

$$\begin{aligned} BC / DC &= (DB + DC) / DC = DB / DC + 1 = 1 / (2 \sin(b/2)) + 1 = \\ &= (1 + 2 \sin(b/2)) / (2 \sin(b/2)) = BH / DT \end{aligned}$$

$$BHC \sim DTC$$

$$BH / DT = ?$$

$$CD / BC = DT / BH = TC / HC$$

$$\begin{aligned} S(ABC) / S(ADC) &= (BH \cdot AC / 2) / (DT \cdot AC / 2) = BH / DT = \\ &= (1 + 2 \sin(b/2)) / 2 \sin(b/2) \end{aligned}$$

$$ACD \sim BDE$$

$$DE / DC = BE / AC = BD / AD = k$$

$$AD / \sin B = AB / \sin ADB = BD / \sin(A/2)$$

$$\begin{aligned} AD / \sin b &= AB / \sin(180 - b - (180 - b) / 4) = \\ &= BD / \sin((180 - b) / 4) \end{aligned}$$

$$BD / AD = \sin((180 - b) / 4) / \sin b = k$$

$$S(BDE) / S(ADC) = k^2 =$$

$$= \sin^2((180 - b) / 4) / \sin^2(b)$$

$$\begin{aligned} (S(ABC) / S(ADC)) / (S(BDE) / S(ADC)) &= \\ S(ABC) / S(BDE) &= \end{aligned}$$

$$= (1 + 2 \sin(b/2)) / (2 \sin(b/2)) //$$

$$// \sin^2((180 - b) / 4) / \sin^2(b) =$$

$$= (1 + 2 \sin(b/2)) \sin^2(b) //$$

$$\sin^2((180 - b) / 4) (2 \sin(b/2))$$

$$\text{ANSWER: } \sin^2((180 - b) / 4) (2 \sin(b/2)) //$$

$$// (1 + 2 \sin(b/2)) \sin^2(b)$$