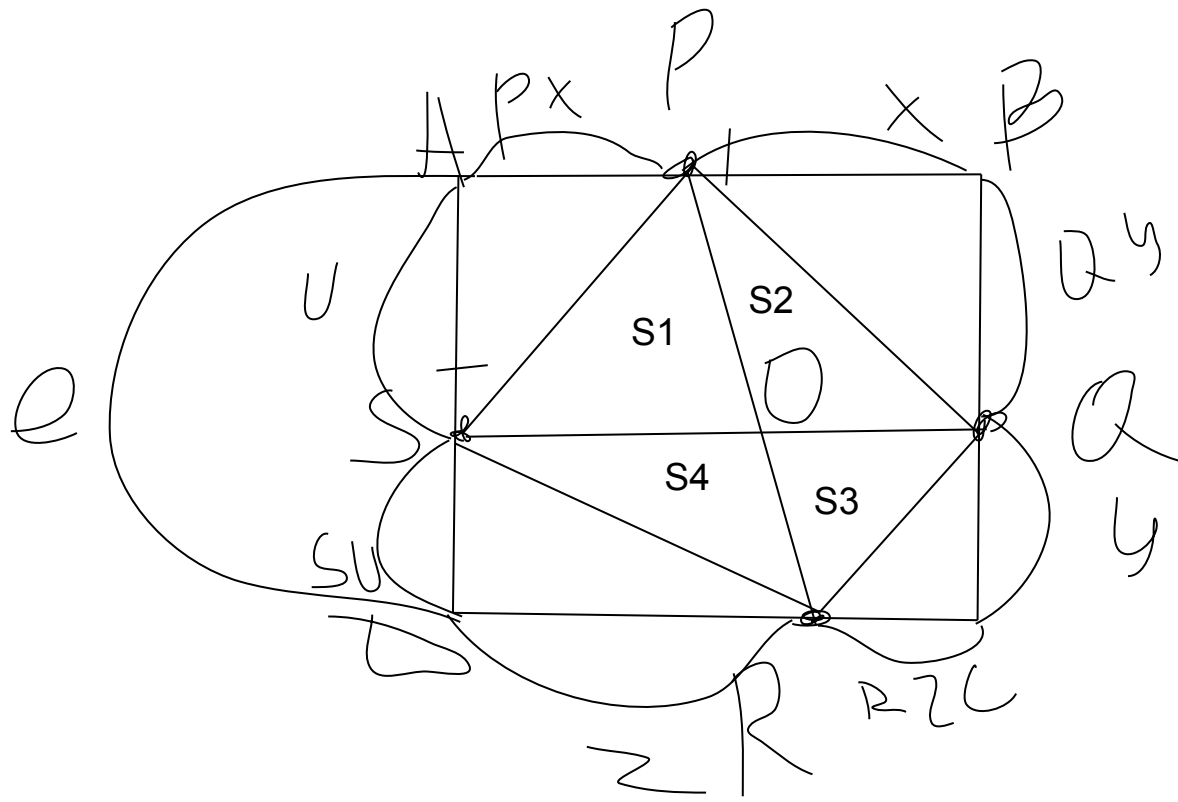


Дан квадрат ABCD. На сторонах AB, BC, CD и AD соответственно выбраны точки P, Q, R, S таким образом, что AP/PB=p, BQ/QC=q, CR/RD=r, DS/SA=s. Отрезки PR и QS пересекаются в точке O. Найти PO/OR.



tip01 достроить отрезки SP, PQ, QR, RS

tip03 нетрудно найти доли в общей площади квадрата S_0
 $S_1+S_2, S_2+S_3, S_3+S_4, S_4+S_1$

tip02 обозначим $S_1=S(SOP), S_2=S(POQ), S_3=S(SDR), S_4=S(PQC)$

tip04 зная S_2 и S_3 - ты сможешь найти PO/OR

$$\begin{matrix} 1*t+1*k+0*g+0*n=a \\ 0*t+1*k+1*g+0*n=b \\ 0*t+0*k+1*g+1*n=c \\ 1*t+0*k+0*g+1*n=d \end{matrix}$$

$$\begin{matrix} 1*t+1*k+0*g+0*n=a \\ 0*t+1*k+1*g+0*n=b \\ 0*t+0*k+1*g+1*n=c \\ 0*t+-1*k+0*g+1*n=d-a \end{matrix}$$

$$\begin{matrix} 1*t+1*k+0*g+0*n=a \\ 0*t+1*k+1*g+0*n=b \\ 0*t+0*k+1*g+1*n=c \\ 0*t+0*k+1*g+1*n=d-a+b \end{matrix}$$

$$\begin{matrix} AP/PB=p \\ BQ/QC=q \\ CR/RD=r \\ DS/SA=s \end{matrix}$$

$$\begin{matrix} px+x=qy+y=rz+z=sv+v=e \\ x=e/(p+1) \\ y=e/(q+1) \\ z=e/(r+1) \\ v=e/(s+1) \end{matrix}$$

$$\begin{matrix} S(ASQB)=(qy+v)e/2 \\ S(ASP)=px*v/2 \\ S(PBQ)=x*qy/2 \\ S_1+S_2=(qy+v)e/2-px*v/2-x*qy/2= \\ =[(qe/(q+1)+e/(s+1))e-pe/(p+1)*e/(s+1)-e/(p+1)*qe/(q+1)]/2= \\ =[qe^2/(q+1)+e^2/(s+1)-pe^2/((p+1)(s+1))-e^2q/((p+1)(q+1))]/2=e^2 \\ 2[q/(q+1)+1/(s+1)-p/((p+1)(s+1))-q/(p+1)(q+1)]/2=e^2W \\ S(PBCR)=(rz+x)e/2 \\ S(RQC)=rzy/2 \\ S(PBQ)=qyx/2 \\ S_2+S_3=(rz+x)e/2-rzy/2-qyx/2= \\ =[(rz+x)e-rzy-qyx]/2=[erz+ex-rzy-qyx]/2= \\ =[re^2/(r+1)+e^2/(p+1)-re^2/((r+1)(q+1))-qe^2/((q+1)(p+1))]/2= \\ =e^2[r/(r+1)+1/(p+1)-r/((r+1)(q+1))-q/((q+1)(p+1))]/2=e^2F \\ S_3+S_4=(su+y)e/2-svz/2-rzy/2= \\ =e^2[s/(s+1)+1/(q+1)-s/((s+1)(r+1))-r/((r+1)(q+1))]/2=e^2J \\ S_4+S_1=e^2[p/(p+1)+1/(r+1)-p/((p+1)(s+1))-s/((s+1)(p+1))]/2=e^2H \end{matrix}$$

$$\begin{matrix} S_1+S_2=e^2W=a \\ S_2+S_3=e^2F=b \\ S_3+S_4=e^2J=c \\ S_4+S_1=e^2H=d \\ S_1=t \\ S_2=k \\ S_3=g \\ S_4=n \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 0 & a^*(-1) \rightarrow 4 \\ 0 & 1 & 1 & 0 & b \\ 0 & 0 & 1 & 1 & c \\ 1 & 0 & 0 & 1 & d \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 0 & ?1 \\ 0 & 1 & 0 & 0 & ?2 \\ 0 & 0 & 1 & 0 & ?3 \\ 0 & 0 & 0 & 1 & ?4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 0 & a \\ 0 & 1 & 1 & 0 & b^*(-1) \rightarrow 1; *1 \rightarrow 4 \\ 0 & 0 & 1 & 1 & c \\ 0 & -1 & 0 & 1 & d-a \end{matrix}$$

$$\begin{matrix} t+k=a \\ k+g=b \\ g+n=c \\ n+t=d \end{matrix}$$

$$\begin{matrix} 1 & 0 & -1 & 0 & a-b \\ 0 & 1 & 1 & 0 & b \\ 0 & 0 & 1 & 1 & c^*1 \rightarrow 1; *(-1) \rightarrow 2; *(-1) \rightarrow 4 \\ 0 & 0 & 1 & 1 & d-a+b \end{matrix}$$

$$\begin{matrix} k=a-t \\ a-t+g=b \\ g+n=c \\ n+t=d \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 1 & a-b+c \\ 0 & 1 & 0 & 1 & b+c \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & 0 & d-a+b-c \end{matrix}$$

$$\begin{matrix} k=a-t \\ g=b-a+t \\ (b-a+t)+n=c \\ n+t=d \end{matrix}$$

$$\begin{matrix} k=a-(a/2+c/2-d/2-b/2) \\ g=b-a+a/2+c/2-d/2-b/2 \\ n=c-b+a-(a/2+c/2-d/2-b/2) \\ t=a/2+c/2-d/2-b/2 \end{matrix}$$

$$\begin{matrix} k=[a-c+d+b]/2=e^2[W-J+H+F]/2=S_2 \\ g=[-a+c-d+b]/2=e^2[-W+J-H+F]/2=S_3 \\ n=[a+c+d-b]/2=e^2[W+J+H-F]/2=S_4 \\ t=[a+c-d-b]/2=e^2[W+J-H-F]/2=S_1 \\ PO/OR=S_2/S_3= \\ =e^2[W-J+H+F]/e^2[-W+J-H+F]= \\ =[-W+J+H+F]/[-W+J-H+F] \end{matrix}$$

$$\begin{matrix} k=a-t \\ g=b-a+t \\ n=c-b+a-t \\ (c-b+a-t)+t=d \end{matrix}$$

$$\begin{matrix} k=a-t \\ g=b-a+t \\ n=c-b-t \\ 2t=-d+c-b+a \end{matrix}$$

$$\begin{matrix} k=[a-c+d+b]/2 \\ g=[-a+c-d+b]/2 \\ n=[a+c+d-b]/2 \\ t=[a+c-d-b]/2 \end{matrix}$$

$$\begin{matrix} k=a-t \\ g=b-a+t \\ n=c-b+a-t \\ t=-d/2+a/2+c/2-b/2 \end{matrix}$$