

Выразите $\cos^3 \varphi$ через тригонометрические функции кратных углов.

$$\cos^3 x = f(\cos 3x)$$

$$(\cos x + i \sin x)^n = (\cos nx + i \sin nx)$$

$$\begin{aligned} (\cos x + i \sin x)^3 &= \cos^3 x + i 3 \cos^2 x \sin x - 3 \sin^2 x \cos x - i \sin^3 x = \\ &= \cos 3x + i \sin 3x \end{aligned}$$

$$(\cos^3 x - 3 \sin^2 x \cos x) + i(-\sin^3 x + 3 \cos^2 x \sin x)$$

$$(\cos^3 x - 3 \sin^2 x \cos x) + i(-\sin^3 x + 3 \cos^2 x \sin x) = \cos 3x + i \sin 3x$$

$$\cos^3 x - 3 \sin^2 x \cos x = \cos 3x \Rightarrow$$

$$\cos^3 x = \cos 3x + 3 \sin^2(x) \cos x = \cos 3x + 3(1 - \cos^2(x)) \cos x =$$

$$= \cos 3x + 3 \cos x - 3 \cos^3(x)$$

$$4 \cos^3(x) = \cos 3x + 3 \cos x$$

$$\mathbf{\cos^3(x) = (\cos 3x + 3 \cos x) / 4}$$

$$\sin 3x = -\sin^3 x + 3 \cos^2 x \sin x = -\sin^3 x + 3(1 - \sin^2)x \sin x = -\sin^3 x + 3 \sin x - 3 \sin^3 x$$

$$4 \sin^3 x = -\sin 3x + 3 \sin x$$

$$\mathbf{\sin^3(x) = (3 \sin x - \sin 3x) / 4}$$