

8. Найдите сумму:

$$\cos \frac{2\pi}{n} + 2 \cos \frac{4\pi}{n} + \dots + (n-1) \cos \frac{2(n-1)\pi}{n}$$

$$z = e^{2\pi i/n}$$

$$\cos x + 2\cos 2x + 3\cos 3x + \dots + (n-1)\cos(n-1)x + n\cos nx$$

$$S_{n+1} = 1 + z + z^2 + z^3 + z^4 + \dots + z^n =$$

$$= 1 + z(1 + z + z^2 + z^3 + z^4 + \dots + z^{n-1}) =$$

$$= 1 + z(S_{n+1} - z^n)$$

$$S_{n+1} = 1 + z(S_{n+1} - z^n)$$

$$S_{n+1} = 1 + zS_{n+1} - z^{n+1}$$

$$S_{n+1}(1 - z) = 1 - z^{n+1}$$

$$S_n = (1 - z^n)/(1 - z) = f(z)$$

$$f(z) = (1 - z^n)/(1 - z)$$

$$(1 + z + z^2 + z^3 + z^4 + \dots + z^{n-1} + z^n)' =$$

$$= 0 + 1 \cdot z^0 + 2z^1 + 3z^2 + \dots + (n-1)z^{n-2} + n \cdot z^{n-1} =$$

$$= 1 + 2z + 3z^2 + \dots + (n-1)z^{n-2} + n \cdot z^{n-1}$$

$$z = \cos(2\pi/n) + i \sin(2\pi/n)$$

$$\operatorname{re}(f'(z)) = 1 + 2\cos(2\pi/n) + 3\cos(4\pi/n) + 4\cos(6\pi/n) + \dots + (n-1)\cos(2\pi(n-2)/n) =$$

$$= 1 + 2\cos(2\pi/n) + 3\cos(4\pi/n) + 4\cos(6\pi/n) + \dots + (n-1)\cos(2\pi(n-2)/n) + n\cos(2\pi(n-1)/n) =$$

$$= 1 + \cos(2\pi/n) + 2\cos(4\pi/n) + 3\cos(6\pi/n) + \dots + (n-2)\cos(2\pi(n-2)/n) + (n-1)\cos(2\pi(n-1)/n)$$

$$(\cos(2\pi/n) + \cos(4\pi/n) + \dots + \cos(2\pi(n-2)/n) + \cos(2\pi(n-1)/n))$$

$$f(z) = (1 - z^{n+1})/(1 - z)$$

$$f'(z) = [(1 - z^{n+1})' \cdot (1 - z) - (1 - z)' \cdot (1 - z^{n+1})] / (1 - z)^2 = [0 - (n+1)z^n \cdot (1 - z) - (-1) \cdot (1 - z^{n+1})] / ((1 - z)^2) =$$

$$= [(1 - z^{n+1}) + (n+1)z^n \cdot (z - 1)] / (1 - z)^2$$

$$\operatorname{re}(f'(z)) = [1 - \cos((n+1) \cdot 2\pi/n) + \dots] / (1 - \cos(2\pi/n))^2$$

$$1 + \cos(2\pi/n) + 2\cos(4\pi/n) + 3\cos(6\pi/n) + \dots + (n-2)\cos(2\pi(n-2)/n) + (n-1)\cos(2\pi(n-1)/n)$$

$$(\cos(2\pi/n) + \cos(4\pi/n) + \dots + \cos(2\pi(n-2)/n) + \cos(2\pi(n-1)/n))$$

$$= [1 - \cos((n+1) \cdot 2\pi/n) + \dots] / (1 - \cos(2\pi/n))^2$$

$$(\cos(2\pi/n) + \cos(4\pi/n) + \dots + \cos(2\pi(n-2)/n) + \cos(2\pi(n-1)/n)) = \operatorname{re}(1 + z + z^2 + z^3 + z^4 + \dots + z^n) =$$

$$\operatorname{re}(1 - z^n)/(1 - z)$$

$$1 + \cos(2\pi/n) + 2\cos(4\pi/n) + 3\cos(6\pi/n) + \dots + (n-2)\cos(2\pi(n-2)/n) + (n-1)\cos(2\pi(n-1)/n) =$$

$$= [1 - \cos((n+1) \cdot 2\pi/n) + \dots] / (1 - \cos(2\pi/n))^2 - \operatorname{re}(1 - z^n)/(1 - z)$$

$$n = 3$$

$$(3-1)\cos \frac{2(3-1)\pi}{3}$$

$$\parallel$$

$$2\cos \frac{4\pi}{3}$$

$$(x^n)' = n \cdot x^{n-1}$$

$$f = \frac{a}{b}$$

$$f' = \frac{a' \cdot b - b' \cdot a}{b^2}$$

Квадратная матрица  
 $a \cdot b \neq b \cdot a$