

$$\sin x \cdot \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x \cdot \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin y \cdot \sin x = \frac{1}{2}[\cos(x+y) - \cos(x-y)]$$

$$\cos x + \cos 2x + \dots + \cos(n-1)x + \cos nx = ? \quad || * \sin(x/2)$$

$$\cos x \sin(x/2) + \cos(2x) \sin(x/2) + \cos(3x) \sin(x/2) + \dots + \cos((n-1)x) \sin(x/2) + \cos(nx) \sin(x/2) = ?$$

$$\frac{1}{2}[\sin(3x/2) + \sin(-x/2) + \sin 5x/2 + \sin(-3x/2) + \sin(7x/2) + \sin(-5x/2) + \dots + \sin(x(2n-1)/2) + \sin(-x(2n-3)/2) + \sin(x(2n+1)/2) + \sin(-x(2n-1)/2)] =$$

$$= \frac{1}{2}[\sin(-x/2) + \sin(x(2n+1)/2)] = \sin([x(2n+1)/2 - x/2]/2) * \cos([x(2n+1)/2 + x/2]/2) = \sin([x(2n+1) - x]/4) * \cos([x(2n+1) + x]/4) =$$

$$= \sin(nx/2) * \cos(x(n+1)/2) = ? * \sin(x/2)$$

$$? = \sin(nx/2) * \cos(x(n+1)/2) / \sin(x/2)$$

$$\sin x + \sin 2x + \dots + \sin(n-1)x + \sin nx = ? \quad || \sin x/2$$

$$-\frac{1}{2}[-\cos(x/2) + \cos(x(2n+1)/2)] = -\frac{1}{2}[\cos(x(2n+1)/2) - \cos(x/2)] = \sin(nx/2) * \sin(x(n+1)/2) = ? \sin x/2$$

$$? = \sin(nx/2) * \sin(x(n+1)/2) / \sin(x/2)$$