

7.13. Докажите, что если $x + iy = (s + it)^n$, то $x^2 + y^2 = (s^2 + t^2)^n$.

$$\begin{aligned}x + iy &= (s + it)^n, \text{ то } x^2 + y^2 = (s^2 + t^2)^n \\|z|(\cos a + i \sin a) &= |w|^n ((\cos b + i \sin b))^n = |w|^n (\cos n(b) + i \sin n(b)) \\(|z|/|w|^n)^2 &= (\cos n(b) + i \sin n(b)) / (\cos a + i \sin a)^2 \\|z|^2/|w|^{2n} &= (\cos 2nb + i \sin 2nb) / (\cos 2a + i \sin 2a) \\|z|(\cos a + i \sin a)^2 &= (|w|^2 (\cos 2b + i \sin 2b))^n\end{aligned}$$

$$\begin{aligned}(a^2 + b^2)(u^2 + v^2) &= (au + bv)^2 + (av - bu)^2 \\|z|^2 |w|^2 &= (a^2 + b^2)(u^2 + v^2) \\z &= a + ib \\w &= v + iu \\zw &= av - bu + i(vb + ua) \\|zw|^2 &= (au + bv)^2 + (av - bu)^2 \\|z|^2 |w|^2 &= |zw|^2 \\|z| |w| &= |zw|\end{aligned}$$

