

7.29. Найдите сумму степеней порядка s всех корней уравнения $z^n = 1$, где s — целое число.

$$z = \cos a + i \sin a$$

$$z^n = 1$$

$$\cos^n a + i \sin^n a = 1 = |1|(\cos b + i \sin b) = \cos b + i \sin b$$

$$b = 0$$

$$a = b + 2Pk$$

$$a = (b + 2Pk)/n$$

$$z_k = \cos a + i \sin a = \cos[(b + 2Pk)/n] + i \sin[(b + 2Pk)/n] = \cos[(2Pk)/n] + i \sin[(2Pk)/n]$$

$$z_k^s = \cos[(s2Pk)/n] + i \sin[(s2Pk)/n]$$

если s кратно n , ----- n

то $\cos[(s2Pk)/n] = 1, \sin[(s2Pk)/n] = 0$

сумма равна n

если s не кратно n ----- 0

$$n = 5$$

$$z_1^s + z_2^s + z_3^s + z_4^s + z_5^s = \cos 0 + i \sin 0 + \cos(s2P/5) + i \sin(s2P/5) + \cos(s4P/5) + i \sin(s4P/5) + \cos(s6P/5) + i \sin(s6P/5) + \cos(s8P/5) + i \sin(s8P/5) = \cos 0 + \cos(s2P/5) + \cos(s4P/5) + \cos(s6P/5) + \cos(s8P/5) + i(\sin 0 + \sin(s2P/5) + \sin(s4P/5) + \sin(s6P/5) + \sin(s8P/5))$$

$$\cos x + \cos 2x + \dots + \cos(n-1)x + \cos nx = \sin(nx/2) \cos(x(n+1)/2) / \sin(x/2)$$

$$\cos(s2P/5) + \cos(s4P/5) + \cos(s6P/5) + \cos(s8P/5) + \cos(10Ps/5) = \sin(5s2P/5^2) \cos(s2P6/5^2) / \sin(2Ps/5^2) = 0$$

$$x = 2Ps/5$$

$$\cos x + \cos 2x + \dots + \cos(n-1)x + \cos nx = \sin(nx/2) \cos(x(n+1)/2) / \sin(x/2)$$

$$\sin x + \sin 2x + \dots + \sin(n-1)x + \sin nx = \sin(nx/2) \sin(x(n+1)/2) / \sin(x/2)$$

$$(\sin(s2P/5) + \sin(s4P/5) + \dots + \sin(s6P/5) + \sin(s8P/5) + \sin(10P/5)) = 0 + \sin(Ps) \sin(x(n+1)/2) / \sin(x/2) = 0$$

7.32-7.35

