

$$\cos nx = T_n(\cos x)$$

$$T_{(n+1)}(x) = 2T_n(x) + T_{(n-1)}(x)$$

$$T_{(n-1)}(\cos x) = \cos((n-1)x)$$

$$T_n(\cos x) = \cos(nx)$$

$$T_{(n+1)}(\cos x) = \cos((n+1)x)$$

$$\cos(x+y) = 2\cos x \cos y - \cos(x-y)$$

$$\cos((n+1)x) = \cos(xn+x) = 2\cos x \cos xn + \cos(xn-x) = 2\cos x \cos xn + \cos(x(n-1))$$

$$T_{(n+1)}(x) = 2T_n(x) + T_{(n-1)}(x)$$

$$\cos((n+1)x) = 2\cos x \cos n^2 + \cos((n-1)x)$$

$$\cos((n+1)x) = T_{(n+1)}(x)$$

$$U_{(n-1)}(\cos x) = \sin nx / \sin x$$

$$U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x)$$

$$U_{(n+1)}(\cos x) = \sin((2+n)x) / \sin x$$

$$U_n(\cos x) = \sin((1+n)x) / \sin x$$

$$U_{(n-1)}(\cos x) = \sin xn / \sin x$$

$$U_{n+1}(x) = 2xU_n(x) - U_{(n-1)}(x)$$

$$\sin((2+n)x) / \sin x = 2\sin x \sin((n+1)x) / \sin x - \sin nx / \sin x = (2\sin(nx+x)\sin x - \sin nx) / \sin x$$

$$\sin(2x+nx) = 2\sin(nx+x)\sin x - \sin nx$$

$$\sin(2x+nx) + \sin nx = 2 \sin((2x+2nx)/2) \sin(2x/2) = 2 \sin(x(n+1)) \sin x$$