

1. Многочлены Чебышёва:

$$T_0(x) = 1,$$

$$T_1(x) = x,$$

$$T_2(x) = 2x^2 - 1,$$

$$T_3(x) = 4x^3 - 3x,$$

$$T_4(x) = 8x^4 - 8x^2 + 1,$$

$$T_5(x) = 16x^5 - 20x^3 + 5x,$$

$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1,$$

$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x,$$

$$T_2(x) = 2xT_1 - T_0(x)$$

$$T_3(x) = 2xT_2(x) - T_1(x)$$

$$U_0(x) = 1,$$

$$U_1(x) = 2x,$$

$$U_2(x) = 4x^2 - 1,$$

$$U_3(x) = 8x^3 - 4x,$$

$$U_4(x) = 16x^4 - 12x^2 + 1,$$

$$U_5(x) = 32x^5 - 32x^3 + 6x,$$

$$U_6(x) = 64x^6 - 80x^4 + 24x^2 - 1,$$

$$U_7(x) = 128x^7 - 192x^5 + 80x^3 - 8x.$$

$$\cos nx = T_n(\cos x)$$

$$U_{(n-1)}(\cos x) = \sin nx / \sin x$$

$$z = |z|(\cos a + i \sin a)$$

$$z^n = |z|^n(\cos na + i \sin na)$$

$$|z| = 1$$

$$z^n = \cos nx + i \sin nx = T_n(\cos x) + i U_{(n-1)}(\cos x) \sin x$$

7.35. а) Докажите, что многочлен

$P(x) = (\cos a + x \sin a)^n - \cos na - x \sin na$
делится на $x^2 + 1$.

б) Докажите, что многочлен

$Q(x) = x^n \sin a - p^{(n-1)} x \sin na + p^n \sin(n-1)a$
делится на $x^2 - 2xp \cos a + 2$.