

$$1) \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots + \frac{1}{(3n-2)(3n+1)} + \dots$$

решение

$$\frac{1}{(3n-2)(3n+1)} = \frac{1}{3} \left( \frac{1}{3n-2} - \frac{1}{3n+1} \right)$$

$$S_n = \dots = \frac{n}{3n+1}$$

$$2) 1 + 2 \cdot \frac{1}{3} + 3 \cdot \left(\frac{1}{3}\right)^2 + 4 \cdot \left(\frac{1}{3}\right)^3 + \dots + (n-1) \left(\frac{1}{3}\right)^{n-2} + n \left(\frac{1}{3}\right)^{n-1} + \dots$$

решение

$$S_n = 1 + 2 \cdot \frac{1}{3} + 3 \cdot \left(\frac{1}{3}\right)^2 + 4 \cdot \left(\frac{1}{3}\right)^3 + \dots + (n-1) \left(\frac{1}{3}\right)^{n-2} + n \left(\frac{1}{3}\right)^{n-1} \quad | \cdot \frac{1}{3}$$

$$\frac{1}{3} S_n = \frac{1}{3} + 2 \cdot \left(\frac{1}{3}\right)^2 + 3 \cdot \left(\frac{1}{3}\right)^3 + 4 \cdot \left(\frac{1}{3}\right)^4 + \dots + (n-1) \left(\frac{1}{3}\right)^{n-1} + n \left(\frac{1}{3}\right)^n$$

$$\frac{1}{3} S_n = \frac{1}{3} + \left[\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] + \left[\left(\frac{1}{3}\right)^3 + 2 \cdot \left(\frac{1}{3}\right)^3\right] + \dots + \left[\left(\frac{1}{3}\right)^{n-1} + (n-2) \left(\frac{1}{3}\right)^{n-1}\right] + \left[\left(\frac{1}{3}\right)^n + (n-1) \left(\frac{1}{3}\right)^n\right]$$

$$\frac{1}{3} S_n = \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots + \left(\frac{1}{3}\right)^{n-1} + \left(\frac{1}{3}\right)^n + \left(\frac{1}{3}\right)^2 + 2 \cdot \left(\frac{1}{3}\right)^3 + \dots + (n-2) \left(\frac{1}{3}\right)^{n-1} + (n-1) \left(\frac{1}{3}\right)^n$$

$$\frac{1}{3} S_n = \frac{1 - \left(\frac{1}{3}\right)^n}{2} + \left(\frac{1}{3}\right)^2 [S_n - n \cdot \left(\frac{1}{3}\right)^{n-1}]$$

$$S_n = [1 - \left(\frac{1}{3}\right)^n - 2n \cdot \left(\frac{1}{3}\right)^{n+1}] \cdot \frac{9}{2} \cdot 2$$

Ответ  $\frac{9}{4}$