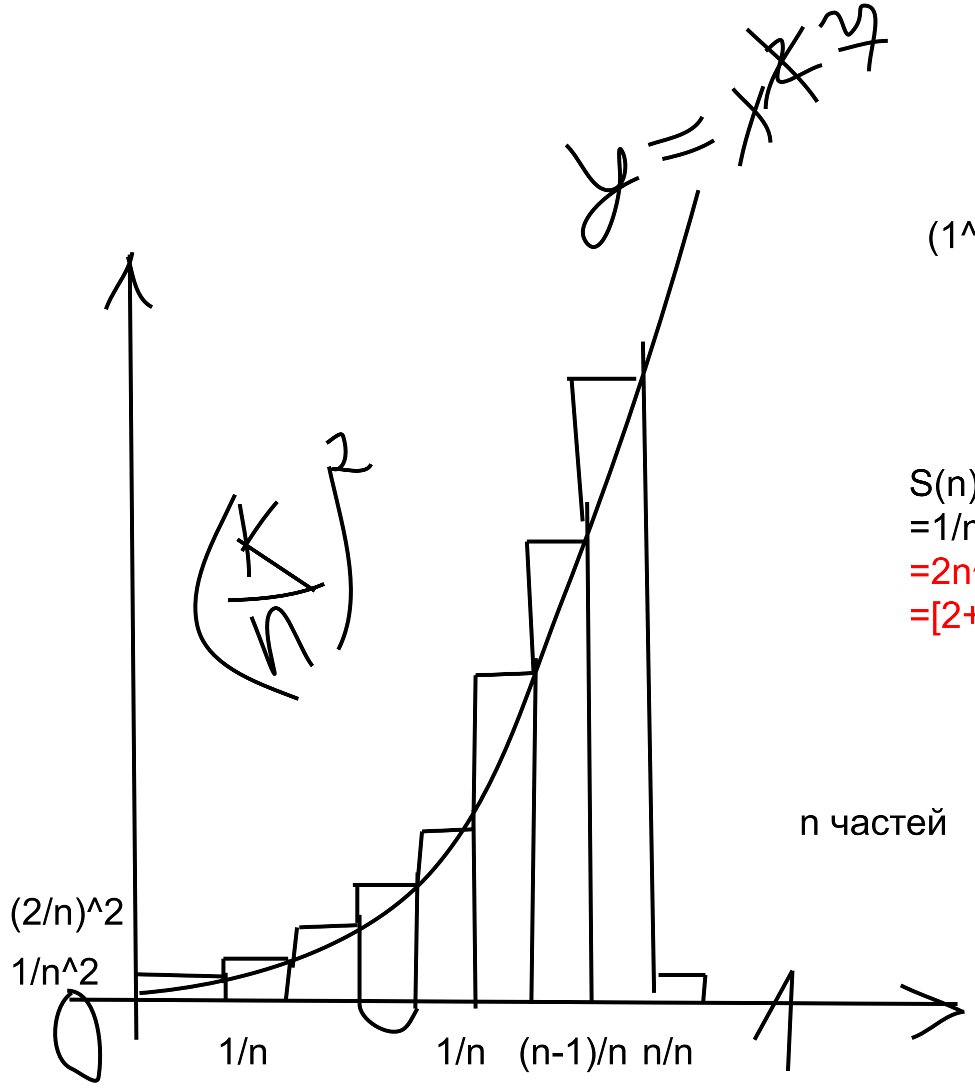
$$\int_0^1 x^2\,dx = \underset{F(x)}{\frac{x^3}{3}}\Big|_0^1 = F(1) - F(0) = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

$$\int x^7\,dx = \frac{x^8}{8} = \frac{1}{8}$$



(1^2+2^2+...+n^2)=n(n+1)(2n+1)/6

S(n)=1/n*((1/n)^2+(2/n)^2+(3/n)^2+....+(n-1/n)^2+(n/n)^2)=
=1/n^3(1^2+2^2+...+n^2)=n(n+1)(2n+1)/6n^3=
=2n^3+3n^2+n / 6n^3=
=[2+3/n+1/n^2] / 6=[2 + 0+0]/ 6= 1/3

n частей

(1^2+2^2+...+n^2)=n(n+1)(2n+1)/6

$$(x^3)' = 3x^2$$

$$x^2 \to \left(\frac{x^3}{3}\right)'$$

1+2+3+...+(n-1)+n=(1+n)*n/2

2^3=(1+1)^3=1^3 + 3*1^2 + 3*1^1 + 1^3
3^3=(1+2)^3=1^3 + 3*2^2 + 3*2^1 + 2^3
4^3=(1+3)^3=1^3 + 3*3^2 + 3*3^1 + 3^3
5^3=(1+4)^3=1^3 + 3*4^2 + 3*4^1 + 4^3
...
n^3=(1+(n-1))^3=1^3 + 3*(n-1)^2 + 3*(n-1)^1 + (n-1)^3
(1+n)^3=(1+n)^3=1^3 + 3*n^2 + 3*n^1 + n^3

2^3+3^3+4^3+5^3+...+n^3+(1+n)^3 = n + 3*(1^2+2^2+...+n^2) + 3*(1+2+3+...+(n-1)+n) + 1^3 +2^3+3^3+4^3+5^3+...+n^3

(1+n)^3 = n + 3X + 3*(1+n)*n/2 + 1^3
X=[(1+n)^3-1-n-3(1+n)n/2]/3=(1+n)[(1+n)^2-1-3n/2)]/3=
=(1+n)(1+n^2+2n-3n/2-1)/3=n(1+n)(n+½)/3=
=n(1+n)(2n+1)/6