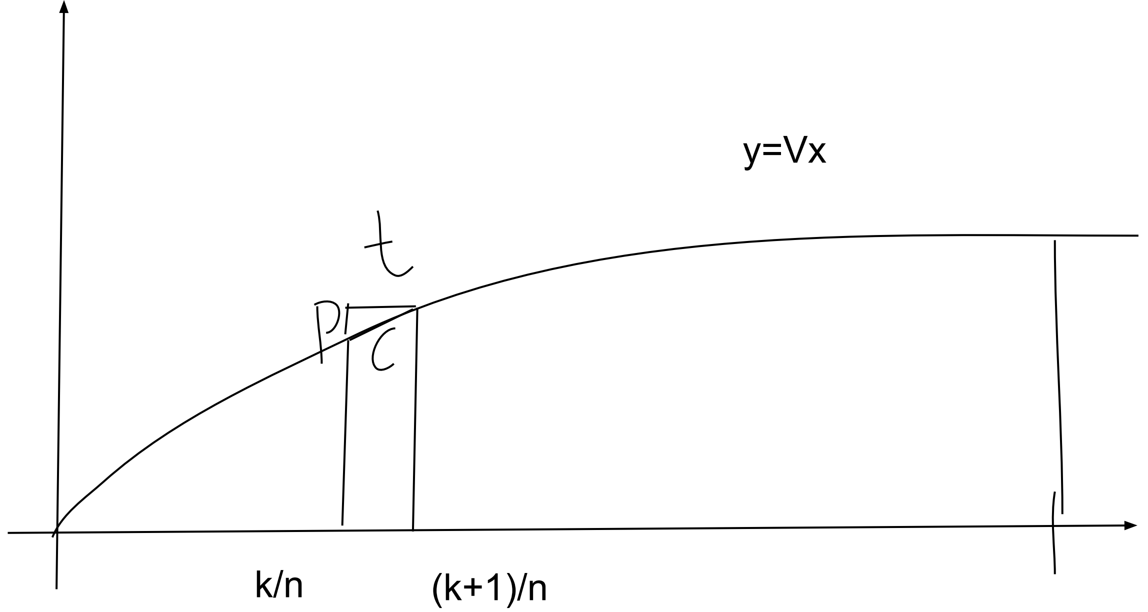
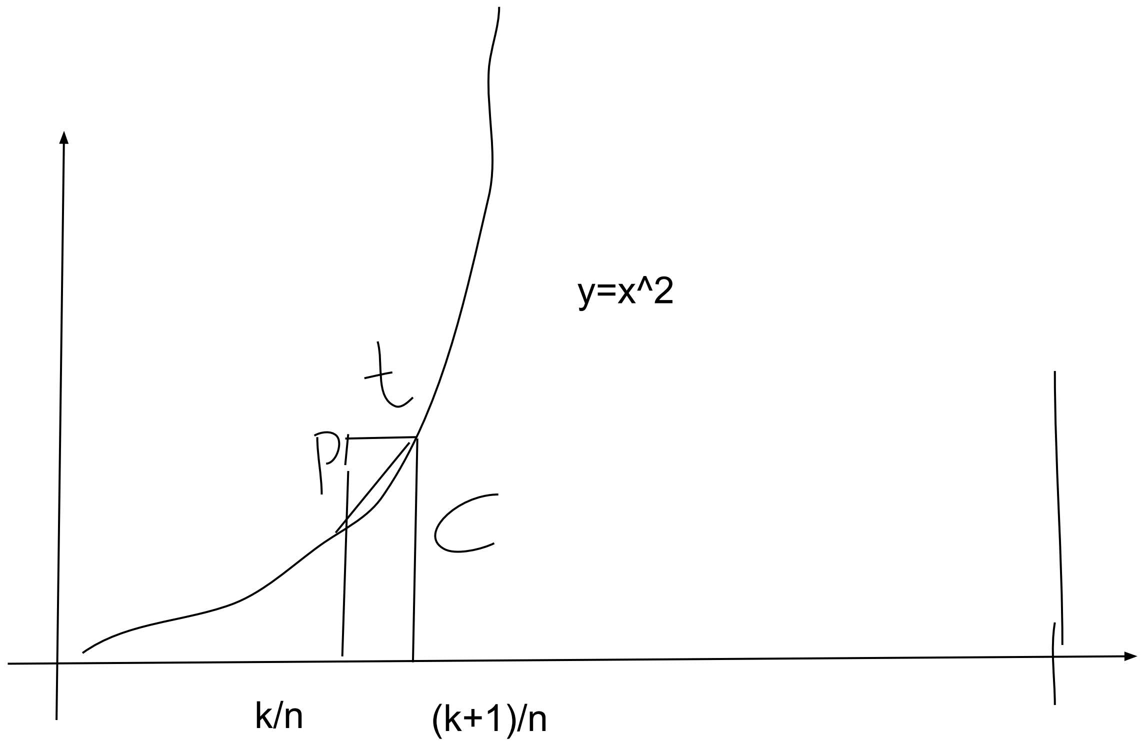


2PS[a;b] y*V((y')^2 + 1)dx



an=1^2+2^2+3^2+...+n^2
a1=1^2
a^2=1^2+2^2

a1=V[1+1]+V[1+9] + V[1+25] + V[1+49]+...+V[1+4+4+1]

t=1/n
p=V((k+1)/n)-V(k/n)
c=V(t^2+p^2)=V((1/n)^2+(V((k+1)/n)-V(k/n))^2)=
=V((1/n)^2+(k+1)/n-2V((k+1)/n)V(k/n))+k/n)=V([1+n(2k+1)]/n^2-2V((k+1)/n)V(k/n)))
=V([1+2nk+n]/n^2-2V((k^2+k)/n))

lim(sqrt(5n^2-4n+1) / (2n-1)) as
n->infinty

t=1/n
p=(k+1)^2/n^2-k^2/n^2
c=V(1/n^2+((k+1)^2/n^2-k^2/n^2)^2)=V[1/n^2+{(k+1)^2-k^2}^2/n^4]=
=V[1/n^2+{k^2+2k+1-k^2}^2/n^4]=V[1/n^2+{2k+1}^2/n^4]=
=V[1/n^2+{4k^2+4k+1}/n^4]=V[(n^2+{4k^2+4k+1})/n^4]=1/n^2*V[n^2+4k^2+4k+1]

k=0: V[n^2+1]
k=1   V[n^2+4+4+1]=V[n^2+9]
k=2   V[n^2+16+8+1]=V[n^2+25]
k=3   V[n^2+36+12+1]=V[n^2+49]

k=n-1   V[n^2+4(n-1)^2+4(n-1)+1]=V[n^2+4n^2-8n+4+4n-4+1]=V[5n^2-4n+1]
k=n   V[n^2+4n^2+4n+1]=V[5n^2+4n+1]

S=1/n^2 [V[n^2+1] + V[n^2+9] + V[n^2+25] + V[n^2+49]+... +V[5n^2-4n+1]]
an=V[n^2+1]+V[n^2+9] + V[n^2+25] + V[n^2+49]+...+V[n^2-4n^2+4n+1]
an-an-1=V[5n^2-4n+1]
bn=n^2
bn-bn-1=1/n^2 - 1/(n-1)^2=(1/n - 1/(n-1))(1/n + 1/(n-1))=(((n-1)- n)/n(n-1))(((n-1)+n)/n(n-1))=(-1/n(n-1))((2n-1)/n(n-1))=-(2n-1)/(n(n-1))^2=(1-2n)/(n(n-1))^2
bn-bn-1=n^2-(n-1)^2=2n-1

V[5n^2-4n+1] / (2n-1) = V[5n^2+4n+1] / (2n-1)]= V[5+4/n+1/n^2] / (2-1/n)]=V5/2

an=V[n^2+1]+V[n^2+9] + V[n^2+25] + V[n^2+49]+...+V[5n^2-4n+1]
an-1=V[(n-1)^2+1]+V[(n-1)^2+9] + V[(n-1)^2+25] + V[(n-1)^2+49]+...+V[5(n-1)^2-4(n-1)+1]

V[n^2+9] - V[(n-1)^2+9]=V[n^2+9] - V[n^2-2n+10]=1/n(V(1+9/n) - V(1-2/n+10/n^2))

V(1+x)~1+1/2x

an=1/n[V[1+1/n]+V[1+9/n] + V[1+25/n] + V[1+49/n]+...+V[5 + (-4n+1)/n]=
=1/n*[1+1/(2n) + 1 + 9/(2n) + 1 + 25/(2n) + 1+49/(2n)+...]=1/(n) [n + 1/2n(1+9+25+49)]

ln(2+sqrt(5))/4=0.360908868

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}).$$

y=x^2   y'=2x
S[0;1]V(1+(y')^2)dx = S[0;1]V(1+(2x)^2)dx =
=S[0;1]V(1+4x^2)dx =   S[0;1]V(1+4x^2)dx
=1/4 (2 x V(1 + 4 x^2) + sinh^(-1)(2 x))|[0;1]
=1/4 (2 x V(1 + 4 x^2) + ln(2x+V(4x^2+1)))|[0;1]
=1/4 (2 V(1 + 4) + ln(2+V(4+1))- ln(1)
=1/4 (2 V(1 + 4) + ln(2+V(4+1))=V5/2 + ln(2+V5)/4

Таким образом:

$$I = x\sqrt{1+4x^2} - I + \frac{1}{2}\ln\left|2x+\sqrt{1+4x^2}\right|$$

$$2I = x\sqrt{1+4x^2} + \frac{1}{2}\ln\left|2x+\sqrt{1+4x^2}\right|$$

$$I = \int\sqrt{1+4x^2}\,dx = \frac{1}{2}x\sqrt{1+4x^2} + \frac{1}{4}\ln\left|2x+\sqrt{1+4x^2}\right|$$