

$$\int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = F(1) - F(0) = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

$$\int x^3 dx = \frac{x^4}{4} = \frac{1}{4}$$

$$1+2+3+\dots+(n-1)+n = (1+n) \cdot n / 2$$

$$2^3 = (1+1)^3 = 1^3 + 3 \cdot 1^2 + 3 \cdot 1^1 + 1^3$$

$$3^3 = (1+2)^3 = 1^3 + 3 \cdot 2^2 + 3 \cdot 2^1 + 2^3$$

$$4^3 = (1+3)^3 = 1^3 + 3 \cdot 3^2 + 3 \cdot 3^1 + 3^3$$

$$5^3 = (1+4)^3 = 1^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 4^3$$

...

$$n^3 = (1+(n-1))^3 = 1^3 + 3 \cdot (n-1)^2 + 3 \cdot (n-1)^1 + (n-1)^3$$

$$(1+n)^3 = (1+n)^3 = 1^3 + 3 \cdot n^2 + 3 \cdot n^1 + n^3$$

$$2^3 + 3^3 + 4^3 + 5^3 + \dots + n^3 + (1+n)^3 = n + 3 \cdot (1^2 + 2^2 + \dots + n^2) + 3 \cdot (1 + 2 + 3 + \dots + (n-1) + n) + 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + n^3$$

$$(1+n)^3 = n + 3X + 3 \cdot (1+n) \cdot n / 2 + 1^3$$

$$X = [(1+n)^3 - 1 - n - 3 \cdot (1+n) \cdot n / 2] / 3 = (1+n) \cdot [(1+n)^2 - 1 - 3n / 2] / 3 =$$

$$= (1+n) \cdot (1+n^2 + 2n - 3n / 2 - 1) / 3 = n(1+n)(n + 1/2) / 3 =$$

$$= n(1+n)(2n+1) / 6$$

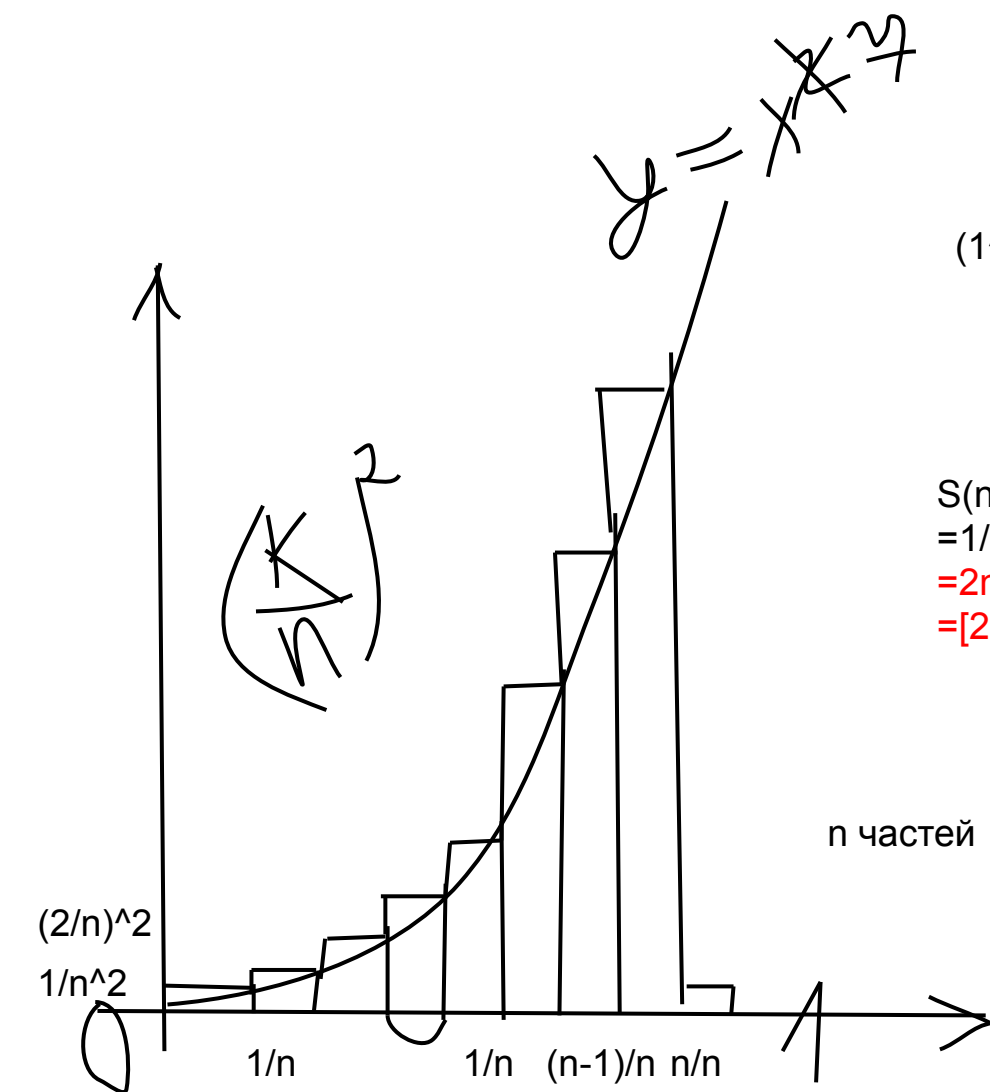
$$(1^2 + 2^2 + \dots + n^2) = n(n+1)(2n+1) / 6$$

$$\begin{aligned} S(n) &= 1/n \cdot ((1/n)^2 + (2/n)^2 + (3/n)^2 + \dots + (n-1/n)^2 + (n/n)^2) = \\ &= 1/n^3 (1^2 + 2^2 + \dots + n^2) = n(n+1)(2n+1) / 6n^3 = \\ &= 2n^3 + 3n^2 + n / 6n^3 = \\ &= [2 + 3/n + 1/n^2] / 6 = [2 + 0 + 0] / 6 = 1/3 \end{aligned}$$

$$(1^2 + 2^2 + \dots + n^2) = n(n+1)(2n+1) / 6$$

$$(x^3)' = 3x^2$$

$$x^2 \rightarrow \left(\frac{x^3}{3} \right)'$$



n частей